

The **variance-gamma distribution**, **generalized Laplace distribution**^[1] or **Bessel function distribution**^[1] is a [continuous probability distribution](#) that is defined as the [normal variance-mean mixture](#) where the [mixing density](#) is the [gamma distribution](#). The tails of the distribution decrease more slowly than the [normal distribution](#). It is therefore suitable to model phenomena where numerically large values are more probable than is the case for the normal distribution. Examples are returns from financial assets and turbulent wind speeds. The distribution was introduced in the financial literature by Madan and Seneta^[2]. The variance-gamma distributions form a subclass of the [generalised hyperbolic distributions](#).

The fact that there is a simple expression for the moment generating function implies that simple expressions for all moments are available. The class of variance-gamma distributions is closed under convolution in the following sense. If X_1 and X_2 are [independent random variables](#) that are variance-gamma distributed with the same values of the parameters α and β , but possibly different values of the other parameters, λ_1, μ_1 and λ_2, μ_2 , respectively, then $X_1 + X_2$ is variance-gamma distributed with parameters $\alpha, \beta, \lambda_1 + \lambda_2$ and $\mu_1 + \mu_2$

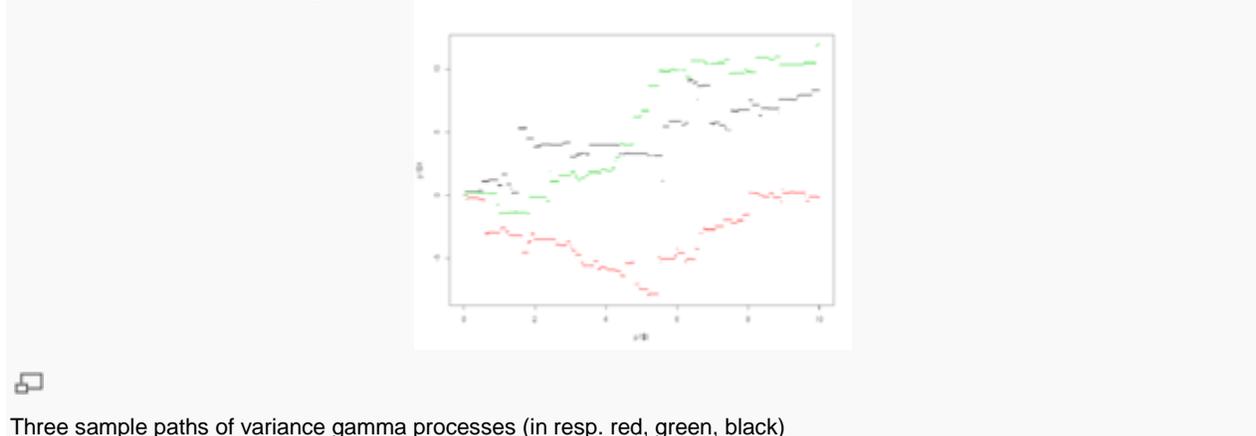
variance-gamma distribution	
parameters:	μ location (real) α (real) β asymmetry parameter (real) $\lambda > 0$ $\gamma = \sqrt{\alpha^2 - \beta^2} > 0$
support:	$x \in (-\infty; +\infty)$
pdf:	$\frac{\gamma^{2\lambda} x - \mu ^{\lambda-1/2} K_{\lambda-1/2}(\alpha x - \mu)}{\sqrt{\pi} \Gamma(\lambda) (2\alpha)^{\lambda-1/2}} e^{\beta(x-\mu)}$ <p>K_λ denotes a modified Bessel function of the second kind Γ denotes the Gamma function</p>
cdf:	
mean:	$\mu + 2\beta\lambda / \gamma^2$

<u>median:</u>	
<u>mode:</u>	
<u>variance:</u>	$2\lambda(1 + 2\beta^2 / \gamma^2) / \gamma^2$
<u>skewness:</u>	
<u>ex.kurtosis:</u>	
<u>entropy:</u>	
<u>mgf:</u>	$e^{\mu z} \left(\gamma / \sqrt{\alpha^2 - (\beta + z)^2} \right)^{2\lambda}$

http://en.wikipedia.org/wiki/Variance-gamma_distribution

Variance gamma process

From Wikipedia, the free encyclopedia



Three sample paths of variance gamma processes (in resp. red, green, black)

In the theory of [stochastic processes](#), a part of the mathematical [theory of probability](#), the **variance gamma process (VG)**, also known as **Laplace motion**, is a [Lévy process](#) determined by a random time change. The process has finite [moments](#) distinguishing it from many Lévy processes. There is no [diffusion](#) component in the VG process and it is thus a [pure jump process](#). The increments are independent and follow a [Laplace distribution](#).

There are several representations of the VG process that relate it to other processes. It can for example be written as a [Brownian motion](#) $W(t)$ with drift θt subjected to a random time change which follows a [gamma process](#) $\Gamma(t;1,\nu)$ (equivalently one finds in literature the notation $\Gamma(t;1/\nu,\nu)$):

$$X^{VG}(t; \sigma, \nu, \theta) := \theta \Gamma(t; 1, \nu) + \sigma W(\Gamma(t; 1, \nu)) \quad .$$

Since the VG process is of finite variation it can be written as the difference of two independent gamma processes ^[1]:

$$X^{VG}(t; \sigma, \nu, \theta) := \Gamma(t; \mu_p, \mu_p^2 \nu) - \Gamma(t; \mu_q, \mu_q^2 \nu)$$

where

$$\mu_p := \frac{1}{2} \sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} + \frac{\theta}{2} \quad \text{and} \quad \mu_q := \frac{1}{2} \sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} - \frac{\theta}{2} \quad .$$

Alternatively it can be approximated by a [compound Poisson process](#) that leads to a representation with explicitly given (independent) jumps and their locations. This last characterization gives an understanding of the structure of the sample path with location and sizes of jumps. ^[2]

On the early history of the variance-gamma process see Seneta (2000). ^[3]

Contents	
[hide]	
1	Option pricing
2	Applications to Credit Risk Modeling
3	Simulation
o	3.1 Simulating VG as Gamma time-changed Brownian Motion
o	3.2 Simulating VG as difference of Gammas
o	3.3 Simulating a VG path by difference of gamma bridge sampling
4	References

[\[edit\]](#)Option pricing

The VG process can be advantageous to use when pricing options since it allows for a wider modeling of [skewness](#) and [kurtosis](#) than the [Brownian motion](#) does. As such the variance gamma model allows to consistently price options with different strikes and maturities using a single set of parameters. Madan and Seneta ^[4] present a symmetric version of the variance gamma process. Madan, Carr and Chang ^[1] extend the model to allow for an asymmetric form and present a formula to price [European options](#) under the variance gamma process.

Hirsa and Madan^[5] show how to price [American options](#) under variance gamma.

Fiorani^[6] presents numerical solutions for European and American barrier options under variance gamma process. He also provides computer programming code to price vanilla and barrier European and American barrier options under variance gamma process.

Lemmens et al.^[7] construct bounds for arithmetic [Asian options](#) for several Lévy models including the variance gamma model.

[\[edit\]](#) Applications to Credit Risk Modeling

The variance gamma process has been successfully applied in the modeling of [credit risk](#) in structural models. The pure jump nature of the process and the possibility to control skewness and kurtosis of the distribution allow the model to price correctly the risk of default of securities having a short maturity, something that is generally not possible with structural models in which the underlying assets follow a Brownian motion. Fiorani, Luciano and Semeraro^[8] model [credit default swaps](#) under variance gamma. In an extensive empirical test they show the overperformance of the pricing under variance gamma, compared to alternative models presented in literature.

[\[edit\]](#) Simulation

Monte Carlo methods for the variance gamma process are described by Fu (2000).^[9] Algorithms are presented by Korn et al. (2010).^[10]

[\[edit\]](#) Simulating VG as Gamma time-changed Brownian Motion

- **Input:** VG parameters θ, σ, ν and time increments $\Delta t_1, \dots, \Delta t_N$,
$$\sum_{i=1}^N \Delta t_i = T.$$
where
- **Initialization:** Set $X(0)=0$.
- **Loop:** For $i = 1$ to N :
 1. Generate independent gamma $\Delta G_i \sim \Gamma(\Delta t_i/\nu, \nu)$, and normal $Z_i \sim \mathcal{N}(0, 1)$ variates, independently of past random variates.
 2. Return $X(t_i) = X(t_{i-1}) + \theta \Delta G_i + \sigma \sqrt{\Delta G_i} Z_i$.

[\[edit\]](#) Simulating VG as difference of Gammas

This approach^{[9][10]} is based on the difference of gamma

representation $X^{VG}(t; \sigma, \nu, \theta) = \Gamma(t; \mu_p, \mu_p^2 \nu) - \Gamma(t; \mu_q, \mu_q^2 \nu)$,

where μ_p, μ_q, ν are defined as above.

- **Input:** VG parameters $\theta, \sigma, \nu, \mu_p, \mu_n$ and time increments $\Delta t_1, \dots, \Delta t_N$,

$$\sum_{i=1}^N \Delta t_i = T.$$

where $i=1$

- **Initialization:** Set $X(0)=0$.

- **Loop:** For $i = 1$ to N :

1. Generate independent gamma

variates $\gamma_i^- \sim \Gamma(\Delta t_i / \nu, \nu \mu_n)$, $\gamma_i^+ \sim \Gamma(\Delta t_i / \nu, \nu \mu_p)$,

independently of past random variates.

2. Return $X(t_i) = X(t_{i-1}) + \gamma_i^+(t) - \gamma_i^-(t)$.

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