Monte Carlo Simulation using Brownian Motion

This uses Brownian Motion popularized by John Hull which is:

\[ \ln\left(\frac{S_t}{S_{t-1}}\right) - \phi((\mu - \sigma/2)T, \sigma \sqrt{T}) \]

\( \ln\left(\frac{S_t}{S_{t-1}}\right) \) is compounded daily return between today and yesterday

approximately distributed \( \phi \) drift menu half variance over time with volatility \( \sqrt{T} \)

This is the square root rule where volatility scales with the \( \sqrt{T} \) of time.

So there is periodic return that is approximately normally distributed. The ratio of the pricing levels are lognormally distributed. The Brownian motion becomes lognormal diffusion process.

The example log return uses a simpler formula of

\[ \ln\left(\frac{S_t}{S_{t-1}}\right) = \alpha + z \sigma \]

This is equal to where alpha is determinitic and the \( z \) standard deviation is stochastic component. The alpha is the drift where it will drift upward with positive expected rate of return which is fixed. The other is a random shock where volatility and \( z \) is a random variable. The \( z \) is scaled by volatility which is a stochastic component process. It could randomly change on each recalculation which is generated by the random \( z \).

How the Monte Carlo is implemented in Excel includes 3 assumptions:

Annual drift (expected return of the stock)=10%
Annual volatility=40%
Initial Stock = $100

Other computed assumptions:
Drift daily=10/252 trading days per year = 0.4%
Volatility daily=40%/\( \sqrt{252} \) because of square root rule=2.52%
Drift (mean)=0.4%-0.5*2.5^2 (subtracting one half the variance) <-with geometric averaging, the volatility over time is eroding the returns
Over time, the process is calculated over each day with a new randomly generated plot.

$N(1,0)$ calculate by NormsInv(Rand()) Excel functions. Rand() gives probability between 0 and 1. NormsInv() translates that into the inverse standard normal cumulative distribution. This gives a value of -3 to 3. This randomized the volatility.

The Log Return can be calculated the the shortened Brownian Motion formula. This is drift+vol*z. Again z is randomly calculated which appears as $N(1,0)$ This add to the random shock which is a function of the volatility.

For the Price, multiply the Initial Price of $100 * EXP(Log Return)$

http://www.youtube.com/watch?v=e79OtCamxD0&feature=relmfu