QUANTITATIVE STRATEGY
Pricing bespoke CDOs: latest developments

- Bespoke CDO pricing is one of the most complicated tasks for correlation desks and CDO investors. Within the Gaussian copula framework, there are numerous ways to account for bespoke portfolios and interpolate the base correlation surface.

- We look at different ways to take into account non standard attachment points, non standard maturities and bespoke portfolios. We show the pros and cons of each method both from a theoretical and numerical point of view.

Smiles of base correlation for a bespoke European portfolio with different equivalent strike methods

Source: SG Credit Research
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For a given bespoke tranche, there are sometimes price differences across investment banks. Indeed, there is no unique pricing framework across dealers. Even if the base correlation Gaussian copula framework seems to remain the rule for most investment banks, the techniques to account for bespoke portfolios are numerous and lead to significant price differences.

The purpose of this article is to analyse the pros and cons of several methods used in the market for bespoke CDO pricing. This includes the methods to interpolate the base correlation smile, to find the correlation assumptions for a bespoke tranche and to account for non-standard maturities. In the first part we discuss how to generate a base correlation surface across maturities and strikes. In the second, we show how to account for bespoke portfolios, i.e. how to determine the correlation assumptions used for a bespoke tranche.

All the methods presented in this report are based on the base correlation approach in the Gaussian copula framework. This framework is detailed in our July 2004 article “Pricing and Hedging correlation products”. Our local correlation model presented in our March 2005 article “Pricing CDO with a smile”, provides an alternative method for bespoke CDO pricing and this will be discussed in a forthcoming article.
Generating the base correlation surface: how to price non-standard tranches on indices

Before pricing bespoke CDO tranches, one needs to generate a base correlation surface for each index (iTraxx, CDX, etc). This is a necessary step because bespoke CDO pricing requires correlation assumptions that are taken from the standard index tranche market. Two issues need to be addressed to generate the correlation surface of a given index:

- How to interpolate the base correlation smile across strikes for a given maturity.
- How to interpolate the base correlation smile across maturities for a given strike

Interpolating the base correlation smile across strikes for a given maturity

The interpolation of the base correlation smile for a given maturity has a strong impact on the pricing of non-standard tranches on indices and also on the pricing of bespoke tranches based on these indices. We compare two methods for interpolating the smile (linear and cubic spline interpolation) and discuss how to extrapolate the smile outside the quoted range.

Linear interpolation across strikes

The first method consists in interpolating linearly between all quoted attachment points. For example, one can interpolate linearly between the 3%, 6%, 9%, 12% and 22% base correlations of the iTraxx index. The problem with the linear interpolation method is that it can generate arbitrage opportunities since the slope of the base correlation smile is not continuous.

Indeed, the market-implied law of losses gives the probability for the loss to be below a given strike, using the base correlation smile as an input (see our March 2005 article “Pricing CDOs with a smile” for more details on this). According to the market-implied law of losses, the probability for the portfolio loss to be below $K$ is:

$$P(\text{Loss} < K) = P(\text{Loss} < K, \rho_K) - \text{Skew}(K, \rho_K) \ast \text{Rho}(K, \rho_K)$$

where $P(\text{Loss} < K, \rho_K)$ is the probability for the loss to be below $K$ using the naive cumulative loss function for the [0-$K$] tranche with the base correlation $\rho_K$ for strike $K$, $\text{Skew}(K, \rho_K)$ is the slope (or skew) of the base correlation smile for strike $K$ and where $\text{Rho}(K, \rho_K)$ is the derivative of the expected loss of the [0-$K$] tranche with respect to the correlation $\rho$. This formula shows that if the skew of the base correlation smile is discontinuous when $K$ increases, the probability for the loss to be below $K$ can be discontinuous. For example, if the slope jumps downwards between $K$ and $K+\epsilon$, the probability for the loss to be below $K+\epsilon$ may be lower than the probability for the loss to be below $K$ which leads to negative probabilities for the loss to be between $K$ and $K+\epsilon$. Even if the slope of the smile is continuous, $P(\text{Loss} < K)$ may decrease between two strikes $K_1 < K_2$ if the slope of the smile changes abruptly between $K_1$ and $K_2$. This also generates arbitrages between tranches.
For example, a 10y 8.9%-9% tranche on the iTraxx S5 index was priced at 68bp on 6 June 2006 with a linear interpolation of the base correlation smile while a 9%-9.1% was priced at 84bp. This is an arbitrage opportunity since it is possible to sell protection on the 9%-9.1% tranche, buy protection on the 8.9%-9% tranche and lock a +16bp positive carry without any risk. These arbitrages are difficult to execute in practice because it is hard to enter such thin tranches. Nevertheless, these arbitrage opportunities are dangerous when pricing squared CDOs or bespoke CDOs. Our February 2006 article “CDO²: new opportunities” details a technique for CDO² pricing. It shows that computing portfolio loss distribution is key to determining CDO² spreads. Jumps in the slope of the correlation smile lead to inconsistent loss distributions and therefore to arbitrages in CDO² prices. In some equivalent strike methods, inconsistent loss distribution also lead to arbitrage in bespoke CDO pricing, for example bespoke tranches with negative spreads. As a result, we favour interpolation methods that are more continuous than the linear interpolation method.

**Cubic spline interpolation across strikes**

The cubic spline interpolation method consists in fitting second-order polynomial functions on each part of the base correlation smile so as to obtain a curve that is continuous and whose slope is continuous. Since the slope of the base correlation smile is continuous, this method generates portfolio loss distributions that are consistent most of the time, i.e. the probability for the loss to be in any given range is always positive. Nevertheless, some arbitrage opportunities may exist even with a cubic spline interpolation if the convexity of the base correlation smile changes too abruptly. For example, a 10y 8.9%-9% tranche on the iTraxx S5 index was priced at 65bp on 6 June 2006 with a cubic spline interpolation of the base correlation smile while a 9%-9.1% was priced at 68bp. This example shows that there is a small arbitrage opportunity between the 8.9%-9% and 9%-9.1% tranches (3bp carry for a risk-free position) but that this arbitrage is “smaller” than the arbitrage with the linear interpolation method (16bp for a risk-free position).
The graph above shows the results of the linear and cubic spline interpolations of the 10y iTraxx Benchmark S5 smile on 6 June 2006. The two interpolation methods give different results for strikes between 3% and 6% and between 12% and 22%. For example, a 5-6% 10y tranche has a 334bp spread with the cubic spline interpolation and a 368bp spread with a linear interpolation. A 12-16% 10y iTraxx tranche has a 20bp spread with the cubic spline interpolation and a 30bp spread with the linear interpolation.

Outside the quoted range
In order to get the whole base correlation smile, one then needs to extrapolate outside the quoted base correlations. The most important part of the correlation smile outside the quoted range is the [0%-3%] part as junior tranches usually have a significant sensitivity to correlation. Several methods are possible for the extrapolation below 3%:

- **Use a rule-of-thumb:** some desks use a constant base correlation between 0% and 3%.
- **Use the bespoke market:** it is possible to calculate some values of the base correlation smile below 3% by using market prices of bespoke tranches on portfolios with a wide spread.
- **Use the tranchelet market:** some dealers started to quote 0-1%, 1-2% and 2-3% tranches called tranchelets because they are very thin. 1% and 2% base correlations can be implied from these tranches. These products are perfect to get a pricing of the base correlation below 3%. However, this market is not liquid and the wide bid-offers prevent players from obtaining accurate information on the shape of the smile below 3%.
- **Use a consistent model for the smile:** more advanced models than the Gaussian copula model are consistent with the whole smile at once and can be used to extrapolate the smile outside quoted ranges. Our local correlation model, presented in our March 2005 article “Pricing CDO with a smile”, provides a way to get the whole smile. Within this framework, under simplifying assumptions, we were able to prove that there should be a smile in the base correlation skew, i.e. the smile should be downward-sloping near 0%.

In our opinion, there is no standard way to extrapolate the smile in the market at the moment.
Interpolating the smile across maturities for a given strike

Pricing CDOs with non-standard maturities, i.e. maturities other than 5y, 7y and 10y, requires interpolating the smile across maturities. This interpolation can be addressed through several techniques. We detail two possible methods here:

- Interpolating the American correlations across maturities. This involves a linear interpolation between quoted flat American correlations.
- Building the zero-coupon correlations across maturities. This requires a bootstrapping of the zero-coupon correlation term structure to fit to market prices of index tranches.

A zero-coupon CDO is a CDO for which all protection payments are paid at the end and there is no spread payment over the life of the product. This convention is not used in practice except for zero-coupon equity pieces. The market convention for bespoke CDOs and index tranches is to use American CDOs, for which protection payments are made as soon as a default occurs and for which spread payments are proportional to the remaining notional in the tranche. The first method detailed here solely considers American CDOs while the second method uses zero-coupon CDOs as building blocks to interpolate the smile across maturities.

Before detailing several techniques for the interpolation across maturities, we first recap the process required for pricing American CDOs through the pricing of zero-coupon CDOs.

**Pricing American CDOs through zero-coupon CDOs**

The most standard algorithm to price CDOs is the recursive algorithm which enables to find the expected loss of the [0-K] tranche for a given time horizon T. The algorithm uses three inputs: spread curves, recovery rates for each name and a correlation assumption. It is much faster than Monte-Carlo simulations but does not provide the timing of defaults: it solely gives the loss distribution at a given maturity T but not before. Nevertheless, it can be used to price American CDO tranches that require knowing the expected loss of the tranche at all times. Pricing a [0-K] American tranche requires pricing the protection leg and the fee leg of the tranche:

- The protection leg of the tranche writes:
  \[
  \text{ExpLoss}^{US}_0(T) = \int_0^T e^{-r(t)} d\text{ExpLoss}^{ZC}_t(t, \rho_{ZC}(t))
  \]
  \[
  \text{ExpLoss}^{US}_0(T) = \text{ExpLoss}^{ZC}_0(T, \rho_{ZC}(t)) + \int_0^T r_t \text{ExpLoss}^{ZC}_t(t, \rho_{ZC}(t))dt
  \]

  where \(\text{ExpLoss}^{ZC}_t(\tau, \rho_{ZC}(\tau))\) is the expected loss in the zero-coupon tranche at time \(\tau\) discounted at time \(t\) and \(\rho_{ZC}(\tau)\) is the zero-coupon correlation with maturity \(\tau\).

- The fee leg of the tranche writes:
  \[
  \text{Spread} \times \text{DV01} = \text{Spread} \times \sum_i e^{-r_i h} \left[1 - \text{ExpLoss}^{ZC}_t(t_i, \rho_{ZC}(t_i))\right] (t_i - t_{i-1})
  \]
  where the times \(t_i\) are the fee payment times.

These two formulas show how to price an American CDO through several zero-coupon CDO pricings.
Interpolating the American correlations across maturities
This method consists in two different steps:

- Compute the flat base correlations that give the right prices for the quoted tranches (5y, 7y and 10y). Since these tranches are American, the base correlations are American.
- Interpolate between these correlations, for example linearly, to obtain American base correlations for all maturities.

For example a 6y 0-6% tranche is then priced with one American base correlation which is the average of the 5y 0-6% base correlation and the 7y 0-6% base correlation.

This method is computationally efficient and easy to implement. Its main drawback is that it only gives American base correlations. Therefore, any pricing that requires zero-coupon base correlation is difficult, for example for CDO², tranche options, zero-coupon equity, etc. It is nevertheless possible to imply zero-coupon base correlations from American base correlations.

Interpolating the zero-coupon correlations across maturities
The second method requires constructing a term structure for zero-coupon correlation. As shown above, an American CDO is equivalent to a portfolio of zero-coupon CDOs expiring at successive maturities (for example every three months). There are two ways to price an American CDO with the recursive algorithm:

- Use a flat base correlation (called the American correlation) to price all the zero-coupon CDOs that constitute the American CDO.
- Price each zero-coupon CDO with its zero-coupon base correlation, assuming a zero-coupon correlation term curve.

The second method requires specifying a shape for the zero-coupon correlation term structure. Among possible shapes, piecewise-constant or piecewise-linear curves are the easiest to implement. Once this shape has been set (for example a piecewise-linear term curve), it is possible to bootstrap each part of the correlation term structure from the quoted market prices of American tranches.

Let us consider the example of the 3% zero-coupon correlation term structure. Using the market price for the 5y 0-3% tranche, one can find the zero-coupon correlation that is consistent with this price, assuming a constant zero-coupon correlation between 0 and 5 years. Then, using the market price for the 7y 0-3% tranche, one can find the 7y zero-coupon correlation that is consistent with this price, assuming a linear zero-coupon correlation term curve between 5y and 7y. Lastly, the 10y 0-3% tranche gives the value of the 10y zero-coupon correlation.
American and zero-coupon correlation term curve for the 3% strike...

... and the 22% strike

The results of these two methods (American and zero-coupon term structures) are shown in the graph above for the 3% and 22% strikes. The graph shows that zero-coupon and American correlations are close but not equal: the difference can reach 0.5% on the 3% strike and 2% on the 22% strike. Pricing a zero-coupon product with an American correlation leads to errors in the valuation: for example, an iTraxx 10y zero-coupon equity piece was priced at 6.8% at the end of April using a 10y zero-coupon correlation (at 9.84%) while it would be priced at 6.57% using an American 10y zero-coupon correlation (at 9.48%), which makes a €23,000 error for a €10m tranche notional.

According to our computations, the prices given by the two interpolation methods are relatively close. For example, for 9y iTraxx tranches, the mark-to-market difference is €4,000 for a €10m 0-3% tranche, €19,000 for a €10m 3-6% tranche and €400 for a €10m 22-100% tranche. These differences are equivalent to American correlation differences of 0.05%, 0.13% and 0.22%, respectively. This result is fairly logical. According to the formula (2) above, the difference between American and zero-coupon expected losses comes from the \( \int_0^T r \cdot \text{ExpLoss}_{0}^{\text{ZC}}(t, \rho_{\text{ZC}}(t)) \, dt \) term which is fairly small for small interest rates.

In conclusion, we recommend using the first method (interpolation of American correlations) for investors who only need to price American CDOs and recommend using the interpolation of zero-coupon correlations if pricing more complicated products like CDO\(^2\) or zero-coupon equity tranches is required.
Accounting for bespoke portfolios

Finding the right correlation assumption for a bespoke portfolio is a tricky task. We start by showing solutions for this problem when the bespoke portfolio has only one reference tranche market (iTraxx tranches, for example). Portfolios that mix European and US names are analysed in the second part of this section.

Bespoke portfolios with one reference index

Pricing bespoke tranches requires finding their equivalent index tranches. Once this equivalent tranche has been determined, pricing the bespoke tranche is easy. It simply consists in applying the standard Gaussian copula pricer to the bespoke portfolio with the base correlations of the equivalent index tranche. Since the equivalent index tranche is not necessarily a standard index tranche (for example, it could be a 6y 4%-5.2% iTraxx tranche), finding the base correlations of this equivalent tranche requires having a base correlation surface for each index. This issue was addressed in the prior section.

Pricing a \([K1-K2]\) tranche with the base correlation approach requires pricing the two equity tranches \([0-K1]\) and \([0-K2]\). We describe here five possible methods to find the equivalent index tranche \([0-K\text{index}]\) for a bespoke equity tranche \([0-K\text{bespoke}]\):

- **Moneyness matching:** the bespoke and index equivalent tranches have the same “moneyness” defined as the ratio between the attachment point and the expected loss of the portfolio. For example, a \([0-8%]\) bespoke tranche is equivalent to a \([0-4%]\) index tranche if the bespoke portfolio expected loss is twice as wide as the index expected loss.

- **Probability matching:** the bespoke and index equivalent tranches have the same probability to get wiped out.

- **Equity spread matching:** the bespoke and index equivalent equity tranches \([0-K]\) have the same spread.

- **Senior spread matching:** the bespoke and index equivalent senior tranches \([K-100]\) have the same spread.

- **Expected loss ratio matching:** the expected loss of the two equivalent equity tranches represents the same percentage of the expected loss of their respective portfolios.

**Moneyness matching**

The moneyness matching approach consists in finding the equivalent tranche that has the same “moneyness” as the bespoke tranche. The moneyness is defined as the ratio between the attachment point and the portfolio’s expected loss. Therefore, the method consists in finding \(K\text{index}\) that verifies:

\[
\frac{K\text{index}}{\text{ExpLoss}_\text{index portfolio}} = \frac{K\text{bespoke}}{\text{ExpLoss}_\text{bespoke portfolio}}
\]

One alternative and fairly equivalent way to compute the moneyness is to compute the ratio between the strike and the portfolio spread. The spread of the portfolio can be interpreted as the average expected loss per year. Therefore, having the same moneyness for two tranches can be interpreted as having the same expected time to get wiped out.
The main advantage of this method is that it is very easy to implement and is immediate. It nevertheless has three major drawbacks:

- If the bespoke portfolio spread is very tight compared to the index spread, the equivalent attachment point is very high in the capital structure and may be more senior than most senior quoted tranches. Let us consider the example of a European bespoke tranche with 100 names trading at 15bp. Any tranche senior to 10.5% has an equivalent attachment point that is higher than 22% in Europe, and its correlation assumption is therefore rather arbitrary.

- More importantly, the moneyness approach does not take into account the dispersion of the bespoke portfolio but only its expected loss. Therefore, it does not distinguish between a 45bp homogeneous portfolio (all CDS trading at 45bp) and a portfolio with all names trading tighter (say at 30bp) except one CDS trading close to default (say at 10000bp). This is a problem for equity tranches because in the first case (homogeneous portfolio), an equity tranche is not very risky while in the second case it is extremely risky.

- The P&L of a bespoke tranche is discontinuous in case of default using the moneyness approach. When a default occurs in the bespoke or index portfolio, the ratio of expected losses of the bespoke and index portfolio changes suddenly. As a result, the equivalent strike and equivalent correlation change, and this creates a discontinuity in the P&L of the tranche. Let us take the example of a €10m [0-4%] tranche on a portfolio of 99 names trading at 30bp and one name trading at 50,000bp. The equivalent strike of this tranche is 3.1% and its correlation is 10.3%. If the risky name defaults and the recovery rate is 40%, the protection buyer receives 0.6% of the overall portfolio notional, which represents €1.5m (15% of the tranche notional). Therefore, the tranche becomes a [0-3.4%] tranche on a portfolio of 99 names at 30bp. The new equivalent strike of this tranche is 3.65% which has a correlation of 12.4%. The mark-to-market of the long protection position in the tranche decreases by €1.575m after the default, and therefore the default generates a negative P&L of -€75,000 (+€1.5m-€1.575m=-€75,000) for the protection buyer. The discontinuity in case of default is a very significant drawback of this method.

**Probability-matching**

For the probability matching approach, the equivalent tranche is the index tranche that has the same probability of being eliminated as the bespoke tranche. This writes:

\[ P\left( \text{Loss}^{\text{bespoke}} \leq K^{\text{bespoke}}, \rho^{\text{index}} \right) = P\left( \text{Loss}^{\text{index}} \leq K^{\text{index}}, \rho^{\text{index}} \right) \]

This method is not completely straightforward as computing the probability of elimination of a bespoke tranche requires a correlation assumption which itself depends on the equivalent strike.

This method works well when taking into account the portfolio dispersion. It is also continuous in case of default. For example, the P&L of a long €10m [0-4%] tranche on a portfolio of 99 names at 30bp and one name at 50,000bp moves only by €3,000 in case of default of the risky name, and the equivalent strikes of the pre-default and post-default tranches are very close (5.12% and 5.08%).
On the other hand, this method has two drawbacks:

- Computing equivalent strikes is numerically difficult when using deterministic recovery rates. Because of this assumption, the loss distribution function is not continuous and subtle numerical schemes are required to create a continuous loss distribution. For example, if all names have a recovery rate of 40% and there are 100 names in the portfolio, the possible losses are multiples of 0.6%. As a result, the loss distribution function is piecewise-constant with discontinuities for each multiple of 0.6%. Furthermore, even with a continuous loss distribution function this method is quite time consuming.

- The probability matching method does not work well for bespoke portfolios with a wide spread. Let’s look at a 5y portfolio with 100 names all trading at 100bp. For the benchmark S5 index, the probability for the loss to be zero after five years was 14%, according to our calculations on 6 June 2006. On the other hand, for any correlation assumption, the probability that bespoke strikes below 1.3% are not hit is lower than 14% because of the wide spread of the bespoke portfolio. As a result, there isn’t any equivalent strike for bespoke strikes lower than 1.3%.

In this case, one solution is to take the 0% base correlation as the correlation assumption for these bespoke tranches. Nevertheless, this example shows that the Main index is not the right reference for wide portfolios and that High Yield index tranches, at least in the US, may be more appropriate.

In the graph below, we show the loss distribution functions for the index and bespoke portfolios computed through the probability-matching approach. The graph shows how the 8% strike of a portfolio with 100 names at 100bp is equivalent to the 2.8% strike of the Benchmark S5 index. It also shows how for small bespoke strikes, no index strike has the same probability of getting hit.

**The probability matching approach finds the strikes that have the same probability of not getting hit**

![Graph showing the loss distribution functions](image)

Source: SG Credit Research

The probability-matching method is slow and does not work well for wide portfolios.
Equity spread matching
The equity spread matching methodology consists in finding the equivalent index equity tranche \([0 - K^{\text{index}}]\) that has the same spread as the bespoke equity tranche \([0 - K^{\text{bespoke}}]\). A correlation assumption is needed to compute the bespoke tranche spread and this correlation assumption itself depends on the equivalent tranche. Therefore, as is the case with the probability-matching approach, a numerical solving is also required.

This method takes dispersion into account. It nevertheless has three major drawbacks:

- **It does not work very well when the bespoke portfolio spread is tight.** In this case, for senior tranches, the tranche spread is quite tight too and generates an equivalent strike higher than most senior quoted tranches. For example, for a European bespoke portfolio of 100 names trading at 15bp, any strike higher than 8.6% has an equivalent strike higher than 22%. If the tranche is too senior, there may be no equivalent tranche at all. Indeed, if the bespoke tranche spread is below the index portfolio spread (which is the spread of the tranche \([0 - 100]\)), then no index tranche matches the spread of the bespoke tranche. This case is rare though: for a European portfolio of 100 names trading at 15bp, this happens only for strikes higher than 47% which are unlikely to be useful in practice.

- **It does not work well for junior tranches on bespoke portfolios with wide spreads.** There is a maximum possible spread for an index tranche. For example, a \([0\%-0.01\%]\) tranche on the iTraxx SS index was priced at 3210bp on 6 June 2006. If the bespoke tranche is too risky, there may not be any index equivalent tranche with the same spread. For example, for a bespoke portfolio with 100 names trading at 100bp, any strike lower than 2.7% does not have any equivalent strike.

- **It is not continuous in case of default.** For example, for a bespoke \([0\%-4\%]\) tranche on a portfolio with 99 names at 30bp and one name at 50,000bp. In case of default of the risky name, the equivalent strike changes from 3.44% to 3.36% due to the default of the risky name and this generates a €5,000 P&L for a long €10m protection position.

Senior spread matching
An alternative way to implement a spread matching approach is to match senior tranche spreads instead of equity tranche spreads. Therefore, it involves finding the index tranche \([K^{\text{index}} - 100\%]\) that has the same spread as the bespoke tranche \([K^{\text{bespoke}} - 100\%]\). This method is not equivalent to the equity tranche spread matching.

- **For tight bespoke portfolios, the senior spread matching approach works better than the equity spread matching approach.** For example for a European portfolio with 100 names trading at 15bp, any tranche lower than 12.75% had an equivalent strike lower than 22% on 6 June 2006.

- **For wide bespoke portfolios, the senior spread matching approach is worse than the equity spread matching approach because it does not find any equivalent strike for a wide range of tranches.** In fact, the spread of a \([K - 100\%]\) tranche on the index is capped by the index spread (which is the spread of a \([0\%-100\%]\) tranche). Therefore, if the bespoke spread is wide, some tranches \([K - 100\%]\) have a spread that is higher than the index spread for any possible correlation assumption and in this case, there is no equivalent strike. For example, for a European portfolio with 100 names at 100bp, any strike lower than 4.9% did not have any equivalent strike on 6 June 2006 with the senior spread matching methodology.

- **It is almost continuous in case of default.** The spread of the senior tranches is less sensitive to default. Therefore this method is far more robust in case of default. Let us take the example of a European bespoke \([0\%-4\%]\) tranche on a portfolio with 99 names at 30bp and one name at 50,000bp. In case of default of the risky name, the equivalent strike changes from 3.44% to 3.36% due to the default of the risky name and this generates a €5,000 P&L for a long €10m protection position.
The expected loss ratio method matches the ratio of the tranche expected loss by the portfolio expected loss.

Expected loss ratio matching

The last method we present here is relatively similar to the moneyness approach. Two tranches are equivalent if they have the same moneyness, defined as the ratio between the expected loss of the equity tranche and the expected loss of the portfolio. Therefore the bespoke tranche $K^{\text{bespoke}}$ and the equivalent tranche $K^{\text{index}}$ verify:

$$\frac{\text{ExpLoss}^{\text{index}}_{\text{portfolio}}}{\text{ExpLoss}^{\text{index}}_{\text{portfolio}}} = \frac{\text{ExpLoss}^{\text{bespoke}}_{\text{portfolio}}}{\text{ExpLoss}^{\text{bespoke}}_{\text{portfolio}}}$$

Looking at expected loss ratios has become standard to analyse the relative value of tranches. We update every week in our Relative Value Trader publication the graphs of expected loss distribution across tranches for iTraxx and CDX indices. For example, on 6 June 2006, we had the following split of expected losses across tranches.

iTraxx 5y expected loss split across tranche, 6 June 2006

Source: SG Credit Research

It works well for most portfolios

The expected loss ratio methodology works well in practice for most bespoke portfolios, either tight or wide. First, it always finds a solution as the expected loss ratio of the bespoke tranche is between 0% and 100% and any ratio between 0% and 100% corresponds to one index tranche. Furthermore, it gives equivalent strikes that are most of the time inside the quoted tranches on indices.

For example, for a European bespoke portfolio with 100 names trading at 15bp, all strikes below 18% have equivalent strikes below 22%, which is the most senior quoted tranche for iTraxx indices.

This methodology has two drawbacks:

- It takes dispersion into account but in a counterintuitive way. For example, if one name widens significantly inside a portfolio, the equivalent strikes given by this method do not change much while they move more with the other methods. Let us take the example of a European portfolio with 100 names trading at 30bp. If one name widens significantly in this portfolio and comes close to default, the equivalent strikes for bespoke strikes below 3% are almost unchanged in the expected loss ratio matching method (less than 0.12% change in the equivalent strike) while the changes are bigger than 0.4% for all other methods. Furthermore, although most equivalent tranches become more junior in this case (which is logical as the bespoke portfolio is riskier if one name comes close to default), some equivalent tranches become more senior with the expected loss ratio matching approach. This is rather counterintuitive in our view.
Another problem occurs when a name comes close to default. There is a jump in the equivalent strike as soon as one name defaults like in the moneyness approach. Let us consider once again the example of a bespoke [0-4%] tranche on a portfolio with 99 names at 30bp and one name at 50,000bp. The equivalent strike changes from 3.9% to 3.3% due to the default of the risky name and this generates a +€80,000 P&L for a long €10m protection position. Once again, this is a significant drawback for this method.

Comparison of all equivalent strike methods

The table below shows the spread of a 3-6% 5y tranche on different portfolios with each method. The differences are very significant for a portfolio with one name close to default and all other names at 30bp (between 84bp for the expected loss ratio matching and 132bp for the equity spread matching). The case of a portfolio with 100 names at 15bp is also interesting. It shows that the equity spread matching approach does not work in this case (spread at -3bp for the 3-6% tranche). The moneyness approach also gives a very tight spread for the 3-6% tranche (2bp).

The expected loss ratio matching approach often gives results that are quite far from the other methods. For example, a 3-6% tranche on a portfolio with 50 names at 15bp and 50 names at 100bp is priced at 301bp with this approach, while it is priced above 320bp with all the other methods.

### Spreads of 3-6% 5y European tranches with each equivalent strike approach

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Moneyness</th>
<th>Probability matching</th>
<th>Equity spread matching</th>
<th>Senior spread matching</th>
<th>Expected loss ratio matching</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 names @30bp</td>
<td>37</td>
<td>44</td>
<td>36</td>
<td>49</td>
<td>52</td>
</tr>
<tr>
<td>99 names @30bp, 1 name @50000bp</td>
<td>129</td>
<td>100</td>
<td>132</td>
<td>105</td>
<td>84</td>
</tr>
<tr>
<td>100 names @15bp</td>
<td>2</td>
<td>12</td>
<td>-3</td>
<td>14</td>
<td>12</td>
</tr>
<tr>
<td>100 names @100bp</td>
<td>1071</td>
<td>1063</td>
<td>1073</td>
<td>1101</td>
<td>1053</td>
</tr>
<tr>
<td>50 names @15bp, 50 names @100bp</td>
<td>334</td>
<td>321</td>
<td>335</td>
<td>330</td>
<td>301</td>
</tr>
</tbody>
</table>

Source: SG Credit Research

When comparing all equivalent strike methods, it is important to look at the range of strikes for which each method works well. In the table below, we computed the bespoke strikes for which each method gives an equivalent strike between 0% and 22%. This gives the range of strikes for which pricing a bespoke tranche is possible. We looked at two examples: one European portfolio with 100 names at 15bp and one European portfolio with 100 names at 100bp.

### Bespoke strike range for which the equivalent strike is between 0% and 22% (European portfolio, 6 June 2006)

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Moneyness approach</th>
<th>Probability matching</th>
<th>Equity spread matching</th>
<th>Senior spread matching</th>
<th>Expected loss ratio matching</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 names @15bp</td>
<td>[0%-10.5%]</td>
<td>[0%-13.4%]</td>
<td>[0%-8.6%]</td>
<td>[0%-12.75%]</td>
<td>[0%-18%]</td>
</tr>
<tr>
<td>100 names @100bp</td>
<td>[0%-66%]</td>
<td>[1.3%-42%]</td>
<td>[2.7%-60%]</td>
<td>[4.9%-42%]</td>
<td>[0%-31%]</td>
</tr>
</tbody>
</table>

Source: SG Credit Research

The expected loss ratio and moneyness matching work for most portfolios

The wider the range, the greater the amount of information the model is able to extract from the quotations of index tranches. A narrow range means that model prices for bespoke CDOs are determined by very few, or just one, index tranches. The table shows that the expected loss ratio matching approach is the most flexible method and that the equity and senior spread matching approaches are quite limited for wide portfolios (strikes lower than 2.7% or 4.9%, respectively, do not have any equivalent strike).

28/07/2006
The graph below shows the base correlation smile for a European portfolio with 50 names at 15bp and 50 names at 100bp with each method. It shows that the moneyness approach and the tranche spread matching approach give relatively close results. The differences between the expected loss ratio matching method and others methods are significant for high strikes.

**Base correlation smile for a bespoke portfolio with 50 names at 15bp and 50 names at 100bp with different equivalent strike methods**

The table below summarizes the advantages and drawbacks of each equivalent strike method.

<table>
<thead>
<tr>
<th>Matching</th>
<th>Moneyness approach</th>
<th>Probability matching</th>
<th>Equity spread matching</th>
<th>Senior spread matching</th>
<th>Expected loss ratio matching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Easy to implement</td>
<td>+</td>
<td>-</td>
<td>=</td>
<td>=</td>
<td>=</td>
</tr>
<tr>
<td>Takes dispersion into account</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>Works well for tight portfolios</td>
<td>=</td>
<td>=</td>
<td>=</td>
<td>=</td>
<td>+</td>
</tr>
<tr>
<td>Works well for wide portfolios</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Continuous in case of default</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>=</td>
<td>-</td>
</tr>
</tbody>
</table>

Source: SG Credit Research

In our opinion, the probability matching and equity spread matching methods are the best approaches because they work for most portfolios and have consistent results when the dispersion of the portfolio increases. Furthermore, the probability-matching approach is consistent with the theoretical results found in our local correlation model (see our March 2005 article “Pricing CDO with a smile” for a description of this model). Using a local correlation model instead of a base correlation mapping process raises very complicated issues in terms of calibration and stability of parameters. This issue will be addressed in a forthcoming article.
Bespoke portfolios with several reference indices

When a bespoke portfolio contains names linked to separate indices (for example European and US names), the equivalent strike methodology needs to be refined to take into account both correlation inputs. For the sake of simplicity, we look here at a Europe/US portfolio. We present here three different methods to take into account several reference indices:

- The separate weighted average method: the correlation assumption $\rho_{\text{bespoke}}$ that is used for the bespoke tranche is a weighted average of the correlations $\rho_{\text{itranxx}}$ and $\rho_{\text{CDX}}$ of the equivalent strikes in the European and American smiles. The Europe and US equivalent strikes are computed separately.
- The joint weighted average method: the correlation assumption $\rho_{\text{bespoke}}$ that is used for the bespoke tranches is a weighted average of the correlations $\rho_{\text{itranxx}}$ and $\rho_{\text{CDX}}$ of the equivalent strikes in the European and American smiles. The Europe and US equivalent strikes are computed jointly.
- The beta method: a beta is assigned to each name in the portfolio. This beta is the squared root of the correlation of the equivalent strikes, i.e. $\beta_{\text{itranxx}}$, $\beta_{\text{CDX}}$. These three methods all rely on the methodology for finding the equivalent strike and correlation in each region (Europe and US).

The separate weighted average method

The separate weighted average method is relatively easy to implement. It assumes that the correlation to use for the bespoke portfolio is a weighted average of the correlations of the equivalent strikes in the Europe and US portfolios. The weights of the US and Europe correlations depend on the percentage of the overall portfolio notional that is made of European and US names.

The bespoke portfolio is first considered as 100% European to find the equivalent European strike $K_{\text{itranxx}}$ and correlation $\rho_{\text{itranxx}}$ and then as a 100% US portfolio to find the equivalent US strike $K_{\text{CDX}}$ and correlation $\rho_{\text{CDX}}$. The bespoke correlation is then defined as the weighted average of the two equivalent correlations:

$$\rho_{\text{bespoke}} = \alpha_{\text{EUR}} \rho_{\text{itranxx}} + (1 - \alpha_{\text{EUR}}) \rho_{\text{CDX}}$$

where $\alpha_{\text{EUR}}$ is the percentage of portfolio notional that is comprised of European names.

The weighted average method has two main drawbacks:

- From a theoretical point of view, considering a mixed Europe/US portfolio as a 100% European portfolio and then a 100% US portfolio is not satisfying. Using this method, the equivalent CDX tranche is not equivalent to the equivalent iTranxx tranche. For example, with the probability matching approach, the probability for the equivalent CDX tranche to get wiped out is not equal to the probability for the equivalent iTranxx tranche to get wiped out.
- This method applies the same correlation to all names when pricing the bespoke tranche which is once again dissatisfying.
The joint weighted average method

The joint weighted average method is a more sophisticated version of the separate weighted average method. It also uses a weighted average of correlations to price the bespoke tranche but it finds the equivalent Europe and US tranches at the same time. Therefore, it requires an algorithm that finds both equivalent strikes jointly.

With the joint weighted average method, the two equivalent strikes $K_{\text{iTraxx}}$ and $K_{\text{CDX}}$ are found jointly so that the bespoke tranche priced with the correlation $\rho^{\text{bespoke}} = \alpha_{\text{EUR}} \rho^{\text{iTraxx}} + (1 - \alpha_{\text{EUR}}) \rho^{\text{CDX}}$ is equivalent both to the iTraxx tranche $K_{\text{iTraxx}}$ and the CDX tranche $K_{\text{CDX}}$.

The joint weighted average gives results that are very close to the separate weighted average method. The joint method ensures that equivalent Europe and US tranches are equivalent between each other, which is satisfying from a theoretical point of view. For example, in the joint probability matching method, the probability for the equivalent European tranche to get wiped out is the same as the probability for the equivalent US strike to get wiped out.

When reference indices (here iTraxx and CDX) are fairly close and when their correlation smiles are relatively close too, the separate and joint weighted average methods give very similar results. Let us take the example of a 5y portfolio with 50 European names at 30bp and 50 US names at 400bp. 0-3% tranches have exactly the same equivalent iTraxx and CDX tranches in the joint and separate methods for all equivalent strike methods. For 0-15% tranches, there are small differences between equivalent strikes (below 0.2% for all methods). Nevertheless, the difference in equivalent correlations between joint and separate methods is less than 0.1%, generating marked-to-market changes of about €2,000 for a €10m tranche notional.

The divergence between joint and separate methods is much higher when the reference indices are very different. Let’s consider a bespoke portfolio mixing European Investment Grade names and US High Yield names. In this case, it is possible to use the iTraxx Main and CDX HY tranches as reference tranches. For a 0-15% tranche with 50 European names at 30bp (linked to the iTraxx index) and 50 US names at 400bp (linked to the CDX HY index), the separate average method gives a 21.2% correlation while the joint average method gives a 20.36% correlation. This generates a €20,000 marked-to-market difference.

We do not recommend using the CDX HY as a reference index because its smile is very volatile and leads to highly volatile valuations for bespoke tranches. Nevertheless, this example shows that the two weighted average methods can give slightly different results.
The beta method

Both weighted average methods have one theoretical flaw: they assign the same correlation to European and US names in the bespoke portfolio. The beta method adjusts for this effect. It prices the bespoke tranche with a beta assigned to each name. This beta is the same for all European names (equal to $\beta^{iTraxx} = \sqrt{\rho^{iTraxx}}$) and for all US names (equal to $\beta^{CDX} = \sqrt{\rho^{CDX}}$). Within the Gaussian copula framework, the assets of the $j^{th}$ company write:

$$A_j = \beta_j X + \epsilon_j \sqrt{1 - \beta_j^2}$$

with $\epsilon_j$ being the specific part of the assets of the $j^{th}$ company. These assets are simulated and a default happens if they fall below a given threshold. This formula replaces:

$$A_j = \sqrt{\rho} X + \epsilon_j \sqrt{1 - \rho}$$

which corresponds to a homogeneous correlation $\rho$.

Using a homogeneous beta equal to the square root of correlation for all names is strictly equivalent to using this correlation. Therefore finding two separate equivalent betas for Europe and the US and then using a weighted average of these betas for the bespoke tranche is perfectly equivalent to using the separate weighted average correlation method.

Here we use two different betas in the bespoke tranche. Therefore, the beta method differentiates between European names and US names. In practice, the differences between equivalent strikes in the beta and weighted average methods are small. As a case study, we look at a 0-15% tranche on a portfolio with 50 European names at 30bp linked to the iTraxx tranches and 50 US names at 400bp linked to the CDX HY tranches. The equivalent strikes given by the beta method and the weighted average method differ by less than 0.2%. In terms of mark-to-market, the differential is less than €10,000 for a notional of €10m. If the 400bp names are linked to the CDX HY tranches, the divergence is bigger both on strikes (1.1% differences between equivalent strikes in the weighted average and beta methods) and on valuations (up to €200,000 differences for a €10m notional).
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Source: SG Credit Research
Credit, Fixed Income & Forex Research

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