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Principles of Principal Components

A Fresh Look at Risk, Hedging, and Relative Value

- Principal Components Analysis (PCA) quantifies movements of the yield curve in terms of three main factors: level, slope, and curvature. In this context, hedging and risk management become a matter of managing exposure to these factors.
- Through yield curve scenarios obtained from PCA, one can set up optimal curve-neutral portfolios, implement a curve view, and perform return attribution.
- Butterfly trades are a popular way to identify and trade rich/cheap sectors of the curve. PCA provides a method by which to structure curve-neutral butterfly trades that isolate the relative value opportunity from any market- and slope-directional bias.

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Executive Summary

Overview

Since our initial publication about Principal Components Analysis (PCA) in the August 1, 1997, issue of *Bond Market Roundup: Strategy*, we have been using the PCA framework extensively in our day-to-day operations for a variety of purposes. There has been strong interest in this approach both internally and from our client base. Needless to say, several interesting questions arose along the way, and as a result, we undertook more detailed studies that helped improve the understanding of PCA and also pointed the way to new applications. In this report, we present a summary of our PCA experience.[1] Our primary focus is on the real-world use of PCA, so a considerable portion of the report is dedicated to practical issues. We also provide a step-by-step guide on how to compute principal components.

There are two main parts to this report. The first part concerns PCA for yield curve modeling, portfolio applications, and risk management. This is the area in which our work on PCA started originally. The essence of PCA is that most yield curve movements can be represented as a combination of three reshaping patterns, called principal components. The most prominent applications are in portfolio risk management and hedging, especially for portfolios with cash flows along the entire yield curve. In that context, the PCs are viewed as the risk factors. Other applications covered in this section include structuring curve-neutral trades, return attribution, and the generation of yield curve scenarios. We discuss in detail how to hedge a portfolio, using the Treasury Index as an example. A number of frequently asked questions are answered at the end of Part 1. These range from whether the PCs need to be updated periodically to whether to use PCs from levels or changes to when the fourth PC can become important.

The emphasis in the second part of the report is in looking for relative value on the curve, and structuring butterfly trades. As such, applications are geared more toward individual trades than portfolio-level decisions. We review different weighting methods for butterfly trades, and propose a new one, based on PCA. Our interactive butterfly model, available on the Internet through Salomon Smith Barney Direct, is also described. Another discussion of whether to use PCs from levels or changes appears in this part, along with extensive simulation results that quantify the profitability of PC-based butterfly trades using a wide variety of model parameter values.

The Appendices contain material that is not central to our discussion but is useful as a reference. Appendix A is a summary of the mathematics of PCs, Appendix B describes the computation of portfolio hedges, and Appendix C is a discussion of various ways to define butterfly spreads and their implications.

[1] Some of the material in this report originally appeared in past issues of our weekly, *Bond Market Roundup: Strategy*. References to these articles are given at the beginning of relevant sections.

Part I. Principal Components for the Entire Yield Curve

- **The conventional approach of using duration to assess risk implicitly assumes perfect correlation between all points on the yield curve with no term structure of volatility. In contrast, the partial duration approach works as if there is no correlation across the curve. Principal Components Analysis (PCA) bridges this gap by taking account of the correlation and volatility structure of the yield curve.**
- **PCA identifies reshaping patterns, called principal components, which explain the variance in the yield curve. Almost all of this variance (99.4%) is captured by the first three PCs, which represent a level shift, slope change, and curvature change.**
- **Portfolio applications of PCA include assessing and hedging curve risk, performing return attribution, and generating realistic yield curve scenarios. In risk management and hedging, PCs are viewed as risk factors, and the objective is to keep the portfolio exposure to PCs within acceptable bounds.**
- **PCs calculated using large data samples give a robust representation of yield curve movements in a wide variety of environments. As such, PCA does not have to be updated periodically to adjust the PCs to the current environment. The volatility, but not the shape, of PCs depends on the horizon for which the PCs are calculated, e.g. four-week PCs are twice as volatile as one-week PCs (square-root scaling).**

Principal Components Analysis for Portfolio Management — Motivation[2]

A number of problems in portfolio management and trading require understanding and quantifying the reshaping of the yield curve. For example, in portfolio immunization, the goal is to maintain the market value of an asset portfolio relative to a liability portfolio under all yield curve movements. Relative-value managers often seek to enhance returns without taking yield curve views. They seek to construct curve-neutral portfolios or trades that will outperform the market. At other times, managers wish to create yield curve exposure. However, to structure optimal portfolios and correctly forecast outcomes, yield curve scenarios must be realistically modeled. Moreover, portfolio managers and traders alike need to understand the risk their positions and portfolios have to the reshaping of the yield curve.

[2] Parts of this section originally appeared in the August 1, 1997, issue of *Bond Market Roundup: Strategy*, Salomon Smith Barney.

Duration provides too little information about yield curve risk . . .

For portfolio managers and traders, the first tool that comes to mind to judge market risk is duration — nominal or effective. Although this is largely still the tool of first resort, investors have realized that it is an incomplete measure. While durations might be matched, portfolios could still be subject to performance mismatches because of yield curve reshaping.[3] To obtain a better understanding of portfolio yield curve risks, many investors have moved to partial-duration analysis, which measures sensitivity to a particular maturity point on the par curve.[4] Indeed, partial durations can add substantially to the understanding of risk gained from simple duration or duration bucketing methods. For example, a ten-year STRIP has a duration of approximately ten years, which is similar in duration to 20-year (roughly) bonds. However, from a yield curve exposure point of view, the STRIP can perform very differently from either a 20-year bond or a ten-year note.[5]

... while partial durations may provide too much information.

Although partial durations certainly enhance understanding, partial durations are, in a sense, too complete. They provide too much information. This is because partial durations define sensitivities to each point on the par curve independently. They do not take into account the correlation between points on the curve. They also do not account for the varying yield volatilities along the curve. For example, if a portfolio is long the two-year sector and short the three-year sector, one position may offset the other to a large degree if appropriately weighted. Therefore, requiring that not only durations, but partial durations match is likely too restrictive. If the same amount of risk to yield curve changes could be achieved with less restrictive constraints, investors will be open to a broader universe of relative-value opportunities.

Principal components analysis assesses yield curve risk more efficiently.

To assess risk more efficiently, we need to succinctly describe the reshaping of the yield curve. One technique that can be used to analyze yield curve reshapings is a statistical approach called *principal components analysis (PCA).* Market participants think of yield curve movements in terms of three components: (1) level shift, (2) slope change, and (3) a curvature change (hump). PCA formalizes this viewpoint. In this section, we provide an overview of PCA and its implications for modeling movements in the yield curve. PCA has been applied by a number of authors to study the yield curve.[6] All of the reported studies, including ours, aim to identify and analyze the main factors driving movements in the yield curve. We also consider various robustness issues, some of which we believe to be addressed in some detail for the first time.

In PCA, the change in the yield curve over a given period is described as a weighted sum of fixed yield curve reshapings. PCA analyzes the covariance matrix of the

[3] Duration measures, of course, are regularly augmented by convexity.

[4] Partial duration is also known as key rate duration and can be defined as sensitivities to either points on the par or spot yield curve.

[5] The ten-year STRIP has more exposure to the ten-year par rate than a ten-year note. This is offset by negative exposures to the seven-year, five-year, three-year, etc. — all shorter par rates. Moreover, the ten-year STRIP has no partial-duration exposure to the 20-year par rate, despite a similar duration to a 20-year bond.

[6] The work on yield curve applications of PCA spans more than ten years, with interest in the subject gaining momentum recently. Some related articles include:

- Robert Litterman and José Scheinkman, "Common Factors Affecting Bond Returns," *The Journal of Fixed Income*, June 1991, pp. 54-61.
- Eduardo Canabarro, "Where Do One-Factor Interest-Rate Models Fail?" *The Journal of Fixed Income*, September 1995, pp. 31-52.
- Eric Falkenstein and Jerry Hanweck, Jr., "Minimizing Basis Risk from Non-Parallel Shifts in the Yield Curve, Part II: Principal Components," *The Journal of Fixed Income*, June 1997, pp. 85-90.

yield changes of different maturity points along the yield curve to optimally determine these fixed reshaping patterns, called *principal components (PCs)*.[7]

For our analysis, we have used the weekly yield changes at 120 constant maturity points (three-month to 30-year maturity range with three-month increments) on the Treasury Model par curve for the period from January 1989 through February 1998 to generate the covariance matrix. This gives rise to 120 PCs. It is important to note that the PCs are uncorrelated with each other by construction. Therefore, none of the PCs can be written in terms of the others. Each PC contributes some unique information not provided by the other PCs.

The first three components explain a high percentage of all yield curve variation.

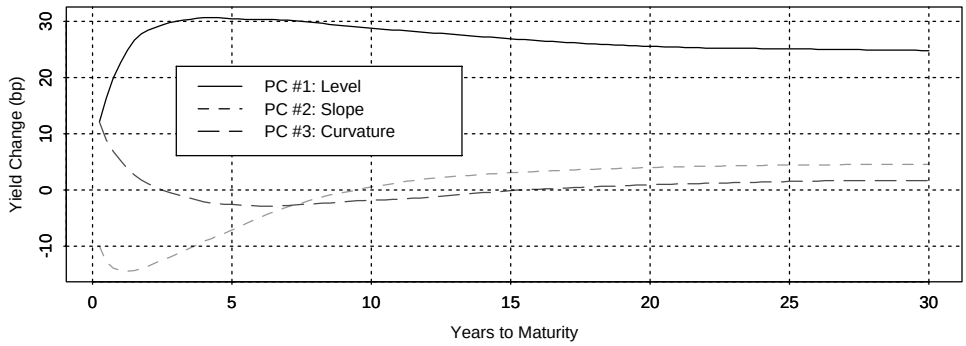
Each PC explains a portion of the total variance. The important PCs are those that explain the highest percentage of the total variance. Sorting the PCs in order of decreasing variance, the first three components explain more than 99.4% of the total variance. Therefore, most movements in the yield curve can be described by using the first three PCs only. Discarding the remaining components omits little. Figure 1 shows the individual and cumulative variances of the first three PCs. Figure 2 shows the first three PCs scaled by the standard deviations of the PCs. The first component accounts for 93.5% of variance; the pattern has positive coefficients at all maturities and consequently represents a level shift. The second accounts for 4.9% of variance; it is negative at the short end and positive at the long end, representing a change in slope. The third accounts for 1.0% and is positive at both ends and negative in the middle: a curvature component.

Figure 1. Variance Explained by the First Three Components

	Component #1	Component #2	Component #3
Standard Deviation	2.77 bp	0.63 bp	0.29 bp
Proportion of Variance	93.5 %	4.9 %	1.0 %
Cumulative Proportion	93.5	98.4	99.4

Source: Salomon Smith Barney.

Figure 2. The First Three Principal Components for a One-Month Horizon Computed from Weekly Yield Changes, Jan 89-Feb 98



Source: Salomon Smith Barney.

The interpretation of the components is intuitive.

We have interpreted the first component as a level shift, despite the fact that it is not a pure parallel shift. Thus, the first component includes the slope and curvature

[7] The fixed patterns are called *loadings* and the random weights are called *principal components (PCs)*. However, in discussion, the terms are used more loosely with the fixed patterns also referred to as PCs, with the principal components called weights.

changes that are correlated with yield level changes. With an upward move in rates (a positive realization of the first PC), the short end of the curve (out to about four years) steepens and the long end flattens, increasing the hump in the intermediate sector of the curve. This observation is in agreement with two often-heard statements: (1) bear-flatteners are more likely than bear-steepeners and (2) the hump (curvature) of the curve becomes more pronounced when yield levels rise. Similarly, for a decline in yields (a negative realization of the first PC), bull-steepeners are more likely than bull-flatteners. These arguments are based on the first component alone, which, due to its high variance, explains the gross behavior of yield curve movements. However, to a lesser extent, the second and third components also contribute to movements in the yield curve, which explains why bear-steepeners and bull-flatteners occur, albeit less frequently.

Applications of Principal Components Analysis of the Yield Curve

There are several ways in which portfolio managers and traders can benefit from yield curve PCA. We focus on four applications: assessment of risk, hedging, return attribution, and constructing yield curve scenarios.

Using PCA to assess risk.

The reshaping scenarios of PCA can be used to efficiently quantify the risk in a portfolio. As discussed in our introduction, duration assumes a perfect 1-for-1 correlation between the changes in yields of different maturities, and is thus prone to misquantify the risk in a portfolio by not accounting for the effects of reshaping. More specifically, since the curve tends to flatten when yields increase and steepen when yields decrease, a portfolio that is solely duration-matched is likely to have a residual directional bias. In contrast, partial duration may find too much risk by not accounting for the covariance structure. However, PCA has determined a set of three factors (reshaping patterns, or PCs) that account for an extremely high proportion of the variance in the yield curve. By comparing a portfolio's sensitivity to changes in each of these PCs to the sensitivity of a benchmark, a good measure of the yield curve risk of a portfolio versus its benchmark can be obtained with only three numbers. Moreover, the three PCs represent independent sources of yield curve risk. Therefore, one can easily obtain the total risk of the portfolio from the individual PC risks and compare this total against allowable risk limits[8].

It is interesting to note that a duration-matched portfolio need not have zero sensitivity to the first PC, the level component, because of the covariance structure of the yield curve. A 10s-30s duration-neutral flattening trade will have positive exposure to the first PC. It will do well in a bear market because 10s-30s tend to flatten in a bear market. The second PC, the slope component, only represents changes in slope that are not correlated with level changes.

Using PCA to structure curve-neutral hedges.

The reshaping patterns determined by PCA can be used to help structure curve-neutral portfolios. If a position has zero exposure to each of the first three PCs, then it will

[8] The total risk of the portfolio is calculated as follows: first the P&L for a 1-standard-deviation move in each of the three PCs is calculated (in the Yield Book). Under a first order approximation, the squares of those three numbers are the variances of independent components of the total P&L. So, the variance of the total P&L is simply the sum of these component variances. Once the variance has been found this way, it is straightforward to compute the value-at-risk at the 5% or 1% levels using a normal distribution assumption.

have zero exposure to any movement in the yield curve that is a linear combination (weighted sum) of the first three PCs (this is to the first order, ignoring the convexity effects of large rate moves). This fact is useful in structuring portfolios. By optimizing an objective function, such as the maximization of dollar return subject to constraints that the exposure to the three PCs is zero, the optimal curve-neutral portfolio can be determined. This portfolio will be more curve-neutral than one structured to simply match duration. The portfolio will likely have a superior value of the objective function than one structured to match partial durations all along the curve because the number of constraints on the portfolio has been reduced to three. Once again, note that the resulting portfolio may not be exactly duration-neutral. We will provide more details about hedging in the next section.

Using PCA for return attribution.

Investors like to know the breakdown of their P&L: how much of the return came from movements in the yield curve (such as level changes, steepening/flattening, etc.), and how much of it came from changes in spread? Furthermore, they are also interested in knowing to what extent their portfolio was mishedged, i.e., did they actually have level exposure, or perhaps a slope bias, beyond what was perceived to exist in the portfolio? PCA provides a unique perspective in quantifying that part of the return attributable to movements in the yield curve. One can easily break down the realized model yield curve movement over the period in question into its first three PCs and a residual reshaping. This is done by fitting the yield curve movement by the three PCs. Then one can calculate the returns from the fitted PCs in the following manner: first, the return under the first PC alone is calculated. Then the return under the combined first and second PCs is calculated and the known return under the first PC is subtracted from the combined return to obtain the part that comes from the second PC. In the next step, the procedure is repeated for all three PCs combined to obtain the return due to the third PC. Finally, the return due to the residual yield curve movement is calculated separately. That part of the realized return that is not explained by the first three PCs and the residual curve movement using this approach is attributed to spread changes and rolling yield.

If a portfolio is hedged against a specific PC, then the return due to that PC is expected to be small. If an investor wants to eliminate level risk by hedging the portfolio against the first PC (level shift), only a small portion of realized return should be from the first PC. A strong steepening or flattening of the curve could manifest itself in the return from the second PC (slope change), especially if the portfolio is mishedged to this component. Similarly, strong fluctuations in the P&L that come from the third PC may indicate a less-than-perfect hedge against curvature changes, unless this exposure is being maintained deliberately to implement a view. By tracking the attribution of the total P&L to the PCs, an investor will have a realistic assessment of whether his/her curve-hedging strategies are meeting their goals, and take corrective action, if necessary, to eliminate unwanted curve exposure.

Using PCA to construct yield curve scenarios.

One of our initial motivations for undertaking the study of PCA was to design realistic yield curve scenarios for use in the Salomon Smith Barney Yield Book™. Investors frequently desire a set of scenarios that cover a range of possible future yield curve outcomes. This set of scenarios could be used to assess performance and risk or to optimize portfolios. We propose the use of the PCs as the basis for

generating realistic scenarios that are consistent with the statistical properties of historical changes in the yield curve.[9]

By itself, the first PC looks like a reasonable scenario, but the second and third do not. Actual yield curve changes are well described by a combination of the first three PCs and are generally dominated by the first PC because of its higher variance. Therefore, to obtain more realistic-looking scenarios, we need to combine the three PCs.

We can create a new set of three scenarios by taking linear combinations of the original PCs without losing any ability to explain variation in the yield curve. Any set of three scenarios that can be obtained from and transformed back to the first three PCs are equivalent from a hedging and immunization standpoint. Therefore, it is possible to have different representations of the yield curve movements that are equivalent to the first three PCs, but have the advantage of looking like scenarios that might actually occur.

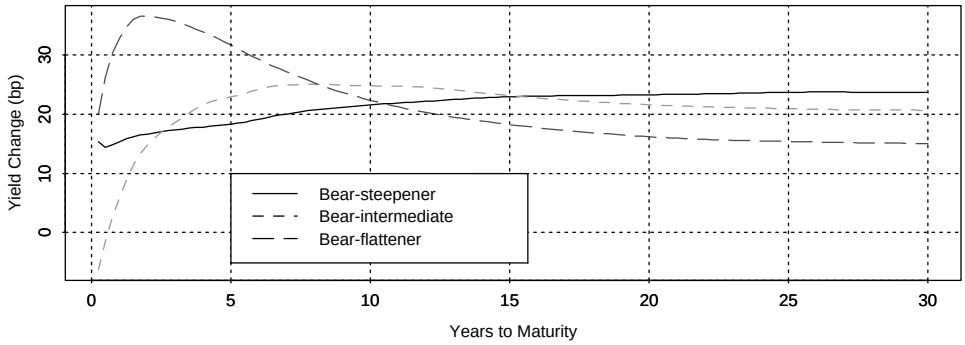
The scenarios in Figure 3 are all equally likely by construction and cover a wide range of movements in the yield curve consistent with historical reshapings of the curve. There are bear- and bull-steepeners, bear- and bull-flatteners, and intermediate scenarios (only the bear scenarios are shown in Figure 3; the full scenarios are negative realizations of the bear scenarios). The scenarios shown in the figure are for a one-month horizon (see below for a discussion of longer time horizons).

***Scenarios based on PCA are available n the* Yield Book.**

Users of Salomon Smith Barney's Yield Book™ have access to scenario files based on our PCA analysis. There are three types of scenario files. In the first type, the scenarios represent the original PCs (see Figure 2). The magnitudes of the rate changes are appropriate to the horizon period based on historical yield volatilities. The names of these scenario files are of the form “pc+horizon.” For example, the PC scenario file for a three-month horizon is named pc3mo. In the second type, the scenarios are linear combinations of the three original PCs that represent a spectrum of possible yield curve movements (see Figure 3). These scenario files are named “cmb+horizon”. For example, the combination file for a one-year horizon is called cmb1yr. Included in each file are both positive and negative realizations of each scenario. Finally, the third type of scenario file contains parallel shifts of ±50bp and ±100bp, as well as combinations of the first two PCs that are centered about these parallel shifts; these files are named bbsf50 and bbsf100. For example, in bbsf50, the first PC is scaled such that it is centered about ±50bp, and the second PC is scaled to one standard deviation (positive and negative), giving rise to four combinations.

[9] A common method for constructing yield curve scenarios uses regression to determine the amount that each point on the curve moves relative to a move in a benchmark maturity (e.g., the 30-year yield). Unfortunately, this method allows only one factor of change in the yield curve. As used here, PCA allows a richer set of yield curves, as reshapings are determined by three factors.

Figure 3. Combination Scenarios for a One-Month Horizon Derived from Principal Components



Source: Salomon Smith Barney.

It is also possible to use PCs in the Yield Book™ to generate scenarios by specifying yield changes at only a number of points. This is useful, for example, when creating economic outlook scenarios based on certain views about specific parts of the curve. Just to illustrate, we may want to create a six-month scenario that depicts the following situation: The Fed hikes the Funds rate by 50bp during the next six months; further rate hikes are expected, so the two-year goes up by 75bp; but because long-term inflation expectations are low, the 30-year barely budges above the +25bp mark. We can have an opinion about those three points, but before we can undertake any analysis, we will also need to know how the entire yield curve reshapes under these constraints. In other words, the specified changes should somehow be interpolated to fill in the unknown points. PCs provide a realistic way of performing this interpolation: the Yield Book™ can automatically compute a combination of the three PCs that satisfy the given yield changes, and since any combination of PCs spans the entire maturity range, we have our scenario. In fact, this is essentially how we periodically generate outlook scenarios that form the basis of our recommended yield curve strategies.

Another application of PCs is to gauge the likelihood of a given scenario. A scenario is broken down into its components (by regression against PCs) and the coefficients of the regression are compared to the standard deviations of the PCs that are appropriate for the horizon of the scenario. For example, if the coefficients represent 1, -3, and 0.5 standard deviation movements in the PCs, then that scenario is a typical bearish scenario with a strong flattening (-3-standard deviations) and a mild change in curvature. This approach allows one to compare a set of proposed scenarios with one another and judge objectively which ones are typical and which ones represent more extreme movements of the curve. The latter could be more appropriate for stress-testing.

A Close Look at Hedging With Principal Components[10]

PCA is a powerful tool for managing portfolios because it allows efficient assessment of yield curve risks. However, investors initially may have difficulty

[10] Parts of this section originally appeared in the February 27, 1998 issue of *Bond Market Roundup: Strategy*, Salomon Smith Barney.

understanding what it means to have exposure to these components and how to effectively use them in practice. In this section, we address the following implementation issues:

Representation of PCs as Trades: We represent the three principal components by three trades. Each trade involves taking long or short positions in three securities. Investors can then translate their portfolio's exposure to the PCs to exposures to these three independent trades. Moreover, a portfolio's exposure can be condensed to positions — long or short — in just these three securities.

Interpreting PC Exposures: We demonstrate that the duration-dollar-weighted yield spreads of these trades can be used to monitor portfolio performance. The dollar exposures to each component can be scaled to represent a 1bp change in these duration-dollar-weighted yields and spreads.

Computation: We look at several methods for computing these component trades, and for calculating a portfolio's curve exposure in terms of the three securities. From this information, a portfolio manager can easily determine what trades must be executed to eliminate any exposure.

Error Estimates: It may be hard to believe that a portfolio such as the Treasury Index could be reasonably replicated with holdings in only three securities. We obtain an indication of errors by testing our hedge over 30 scenarios. The hedge portfolio performs well, and we conclude that using principal components computed with data from a long time period is best.

For comparison, we also assess the performance of a partial-duration-matched portfolio using seven securities.

Selection of the Hedge Securities: We analyze portfolio hedges constructed from several sets of three securities. We also assess the effectiveness of using Treasury futures as the hedge vehicles. We conclude that the best hedges occur when the three securities are "well-spaced" along the curve.

Representing Principal Components as Trades

Duration has always been the initial measure to start with when assessing the risk of a portfolio or the risk of a portfolio versus its benchmark. At first, modified (nominal) duration[11] was used and later this was extended to effective duration.[12] A mismatch in duration is easy to interpret — a portfolio is either long or short the market. Eliminating this exposure by shortening or lengthening is easy to do.

Partial durations were the next step in measuring yield curve exposure. Partial durations (or key rate durations) separate a portfolio's total duration into sensitivities to yield changes at several points along the curve. Thus, partial durations recognize that yields along the curve rarely, if ever, change by the same amount. Again, interpretation is relatively easy — a portfolio is either long or short each maturity sector along the curve. It is easy to eliminate any exposure at each partial duration maturity — simply by selling/buying the appropriate amount of notes of the same

[11] Price sensitivity to a 1bp change in the yield of the security.

[12] Price sensitivity to a 1bp parallel shift of the yield curve.

maturity. Moreover, selling five-year notes to eliminate mismatch in the five-year partial will not change exposures at any of the other six partial durations.

PCA provides a more efficient way to quantify exposure to the yield curve because it considers the correlations and variances of different points along the yield curve. The analysis suggests that only three factors of variation are sufficient to capture most variations in the yield curve. However, understanding the exposure to each PC and identifying the trades to eliminate that exposure is not so easy. For example, suppose a portfolio (versus its benchmark) has exposure to the first PC — a level shift — such that it underperforms if rates decline. Despite the fact that the first PC is not a simple parallel shift of the curve (as it includes slope and curvature changes that are correlated with moves in market direction), a first reaction to eliminate the exposure would be to extend duration by, say, buying five-year notes. However, such a solution would eliminate the exposure to component No. 1, but likely create new exposures to components Nos. 2 and 3. Therefore, identifying the hedge for PC exposures is more complicated.

Remember that the three PCs were determined analytically to be independent (not correlated) factors of variation in the yield curve. Therefore, to represent each component, we will identify a trade that has exposure to one PC but zero exposure to the other two PCs. That is, we identify **three Independent Trades** to interpret and hedge the three independent PCs. Since there are three factors of variation, three securities should be sufficient to define these trades. Using 2s, 10s, and 30s as our three hedge securities, the three independent trades we have identified are shown in Figure 4. These trades have intuitive meanings and are only slightly more complicated than we might have hoped.[13]

Figure 4. Independent Trades Representing the Three Principal Components, Expressed as a Percentage of Duration Dollars

	Independent Trade One Level	Independent Trade Two Slope	Independent Trade Three Curvature
Two-Year	-15%	100%	-25%
Ten-Year	-45	-26	100
30-Year	-39	-87	-88
	-100%	-13%	-13%

Source: Salomon Smith Barney.

1. **The Independent Trade for PC No. 1, an increase in yield level, is to be short the market**. The trade is short all three securities. The amounts are chosen so that the trade has zero exposures to PC Nos. 2 and 3.
2. **The Independent Trade for PC No. 2, an increase in the slope of the curve, is a steepening trade**. The trade is to be long the short end of the curve and short the long end of the curve. This trade is a combination of a 2s-10s steepening trade and a 2s-30s steepening trade, with more duration dollars in the latter.

[13] It is important to recognize that simplification of these trades — for example, into a one-security level trade, or a two-security 2s-10s slope trade — would create trades that no longer have zero exposure to the other components. Then one could no longer simply add up the trades to find the overall portfolio hedge.

3 **The Independent Trade for PC No. 3, a decrease in curvature, is a butterfly trade**. The trade is long the center (10s) and short the wings (2s and 30s).

In Figure 5, we show the size of each trade that will provide a $1 million profit (exposure) to a one-standard-deviation move (over a one-month period) in each component.[14] Again, these trades each have exposure to only one PC. The trades need not be either cash- or duration-dollar-neutral. However, notice that they are close to duration-dollar-neutral.

Figure 5. Independent Hedge Trades for $1 Million Exposure to a One-Month One-Standard-Deviation Move in Each Component (Market Value in $ Thousands), 27 Feb 98

	Analytic Solution			Yield Book Optimization Solution		
	Independent Trade One — Level					
	EDUR$	**Weight**	**Market Value**	**EDUR$**	**Weight**	**Market Value**
Two-Year	-$5,686	15.42%	-$30,537	-$5,768	15.38%	-$30,976
Ten-Year	-16,735	45.40	-22,033	-17,021	45.38	-22,409
30-Year	-14,442	39.18	-10,600	-14,715	39.24	-10,800
	-$36,863	**100.00%**		**-$37,503**	**100.00%**	
	Independent Trade Two — Slope					
	EDUR$	**Weight**		**EDUR$**	**Weight**	
Two-Year	$57,281	100.00%	$307,633	$57,051	100.00%	$306,395
Ten-Year	-14,480	-25.28	-19,064	-15,010	-26.31	-19,762
30-Year	-49,055	-85.64	-36,004	-49,584	-86.91	-36,392
	-6,254	**-10.92%**		**-$7,543**	**-13.22%**	
	Independent Trade Three — Curvature					
	EDUR$	**Weight**	**Market Value**	**EDUR$**	**Weight**	**Market Value**
Two-Year	-$71,838	-25.14%	-$385,813	-$71,854	-25.25%	-$385,899
Ten-Year	285,769	100.00	376,235	284,535	100.00	374,611
30-Year	-248,715	-87.03	-182,544	-249,858	-87.81	-183,383
	-34,784	**-12.17%**		**-$37,177**	**-13.07%**	

Source: Salomon Smith Barney.

Computing Portfolio Hedges

We provide a brief overview of some procedures for computing these independent trades and replicating portfolios (or hedges) once a set of three hedge securities has been selected. We defer the details to the Appendix.

We rely on the scenario analysis and optimization features of the Yield Book™ for at least part of our calculations. As we pointed out before, we have provided scenario files in the Yield Book™ that represent positive and negative realizations of the three PCs. There are scenario files for different horizons as the components are scaled to the standard deviation of curve changes appropriate for the length of the horizon (one-, three-, and six-month). Moreover, we have created sets of yield curve scenarios by combining the PCs. These are the combination scenarios (or the combos). The scenarios represent equally likely outcomes and provide a spectrum of

[14] All dollar returns referred to herein are calculated using a zero-day horizon.

different types of yield curve movements. These scenarios are also available on the Yield Book™ scaled for different horizons.

We suggest two methods for computing the Independent Trades that represent the three PCs. The first is an analytic solution. If the sensitivity of each hedge security to each component is known, it is straightforward to solve for the weights for each independent trade analytically by linear algebra. The second method uses the optimization feature of the Yield Book™ to easily obtain the weights for each independent trade. Both procedures arrived at nearly identical solutions to the examples we have tried.

For computing replicating portfolios (or hedges), we considered four methods: (1) calculate the hedge from the Independent Trades; (2) utilize a direct analytic solution using linear algebra; (3) optimize in the Yield Book™ by "maxmin" on the return differences over the PC scenarios; and (4) optimize in the Yield Book™ by "maxmin" on the return differences over the combo scenarios. We tried all methods for determining the replicating portfolio of three securities for the Treasury Index. We found that all four methods produced similar results. For ease of implementation, we currently recommend using the method of optimization over the PC scenarios.

Hedging the Treasury Index

In Figure 6, we show the performance of the replicating portfolio versus the Index under the combination scenarios for various horizons[15]. Not surprisingly, the performance is quite solid. Tracking performance deteriorates as the horizon increases. This is the result of slight convexity mismatches between the Index and the replicating portfolio.

Figure 6. Performance of Principal Component One-Month Hedge Under Combination Scenarios for Various Horizons (Versus the Treasury Index)

	One-Month	Three-Month	Six-Month
Bearsteep	-0.02 bp	0.02 bp	0.08 bp
Bearintrm	0.02	0.08	0.18
Bearflat	-0.05	-0.10	-0.16
Bullflat	0.09	0.21	0.39
Bullintrm	0.07	0.18	0.33
Bullsteep	0.04	0.07	0.11

Source: Salomon Smith Barney.

Estimation of Tracking Error — In-Sample Versus out-of-Sample Performance

The solid performance results in Figure 6 are comforting, but they are not convincing evidence of the ability to match the performance of the Index with just three securities. This is because the combination scenarios were generated from the same PCs that were used to determine the hedges: this constitutes in-sample testing. A more realistic measure of hedge performance requires out-of-sample testing.

[15] The Appendix contains details of how this hedge can be obtained.

To perform an out-of-sample test, we examine the performance over four other sets of combination scenarios. The components that the combos were based on were calculated from data from four distinct two-year or three-year periods over the last nine years. This combined set of 24 (12 bull and 12 bear) scenarios covers a wide range of yield curve reshapings. Each of the scenarios is scaled by a factor appropriate to a three-month horizon. We add to the 24 scenarios the six scenarios from the original components derived from data from the full nine-year period 1989 through February 1998) for a grand total of 30 scenarios.

For each period shown in Figure 7, we constructed a hedge based on the PCs calculated from data from that period. Then, we tested its performance under the 24 scenarios from the other time periods. As expected, when the scenarios are generated from components from a different time period than that used for the optimization, return performance deteriorates. However, performance is best for the hedge constructed from the longest period, 1989–1998. The worst tracking performance for this solution (hedge minus index over 24 scenarios) was -5bp and the best +4.9bp, with a standard deviation of errors of 2.6bp. In contrast, for hedges created from scenarios over shorter periods, such as 1989–90, we find larger tracking errors when they are analyzed over other time periods. This led us to conclude that PCs calculated from data from a long time period are best. The shorter periods likely do not contain as full a spectrum of yield curve reshapings and the components may reflect changes in relative value particular to the period.

Figure 7. Performance of Hedges Versus Treasury Index Under Various Three-Month Scenarios

	Hedge		Index		Diff.				Quartile				
Sub-Period for Hedge	Eff. Dur.	Eff. Conv.	Eff. Dur.	Eff. Conf.	Eff. Dur.	Eff. Conv.	Std. Dev.	Mean	Min	25 %	50 %	75 %	Max
1989-98	5.09	0.606	5.10	0.594	-0.01	0.012	2.63 bp	0.08 bp	-5.09 bp	-1.55 bp	0.09 bp	1.80 bp	4.89 bp
1989-90	5.10	0.527			0.00	-0.067	7.30	-0.24	-13.21	-5.41	-0.53	4.67	13.45
1991-92	5.01	0.564			-0.08	-0.030	4.16	-0.18	-6.54	-4.04	-0.08	3.79	5.92
1993-94	5.10	0.624			0.00	0.030	3.79	0.17	-8.16	-1.21	0.22	1.68	8.07
1995-97	5.13	0.653			0.03	0.059	4.62	0.32	-8.89	-1.94	0.40	3.10	8.56

Source: Salomon Smith Barney.

Selecting the Hedge Securities

In our example, we have used 2s, 10s, and 30s as the hedge securities. Why did we chose these issues and are they the best? Would other alternatives produce similar results?

We used the 30 curve scenarios to test the stability of tracking performance using various sets of three securities chosen from 2s, 3s, 5s, 10s, 25s, 30s, and bond futures. In addition, we show the return performance for the partial-duration matched portfolio of seven securities with one, two, three, five, ten, and 30 years to maturity. A summary of the results is shown in Figure 8. Most hedges demonstrate very good tracking performance. We make several observations:

- Performance is better when the three securities are “well-spaced.” The worst performances occurred when the short-end of the curve or the long-end of the curve were not represented. If the selected issues are too close together in maturity, the

correlation between their performance will be high. Then the issues will not as completely represent the distinct sources of variation in the yield curve.[16]

- Replacing 30s with 25s improved the tracking error. It also reduced the convexity of the hedge position. Likely, 25s hedge the bond sector better than combinations of 5s/10s and 30s on average.
- The partial-duration-matched portfolio of seven securities did not provide superior performance to the three security hedges under these scenarios. One possible reason is that the methodology for computing partial durations assumes straight-line interpolation of the yield curve between the partial maturities. A second reason may be the use of 30s instead of 25s.
- The hedges with futures performed comparably to hedges with cash securities. However, these hedges do not provide the smallest tracking errors. Possible reasons include the fact that bond futures represent 17-year bonds, leaving 13 years of the curve beyond the hedge instrument. In addition, ten-year and five-year futures actually represent a separation of only 2.5 years along the curve.

Figure 8. Selection of Hedge Vehicles — Performance of Hedges Versus the Treasury Index Under Various Three-Month Scenarios

	Hedge		Index		Diff				Quartile				
	Eff. Dur.	Eff. Conv.	Eff. Dur.	Eff. Conv.	Eff. Dur.	Eff. Conv.	Std. Dev.	Mean	Min.	25 %	50 %	75 %	Max
2s, 5s, 25s	5.10	0.644	5.10	0.594	0.00	0.050	1.29 bp	0.26 bp	-2.51 bp	-0.15 bp	0.30 bp	0.70 bp	3.01 bp
3s, 10s, 25s	5.07	0.606			-0.03	0.012	1.54	0.07	-2.70	-1.16	0.04	1.23	3.10
PDUR$ Hedge	5.10	0.597			0.00	0.003	1.58	-0.01	-3.10	-1.08	0.00	1.15	3.10
2s, 5s, 30s	5.12	0.710			0.02	0.116	1.65	0.58	-3.04	-0.16	0.52	1.91	3.36
2s, 10s, 25s	5.08	0.572			-0.01	-0.022	1.97	-0.09	-4.50	-1.74	-0.05	1.51	4.08
2s, 10s, 30s	5.09	0.606			-0.01	0.012	2.33	0.08	-4.97	-1.34	0.07	1.51	4.76
2s, 10s, 30s Futures	5.08	0.409			-0.02	-0.185	2.51	0.36	-6.31	-0.56	0.49	1.67	6.46
5s, 10s, 30s Futures	5.07	0.623			-0.03	0.029	2.71	-0.95	-6.75	-2.75	-1.00	0.49	4.21
5s, 10s, 30s	5.13	0.755			0.04	0.161	2.80	0.81	-5.25	-0.52	0.79	2.12	6.84
2s, 5s, 10s	5.07	0.444			-0.03	-0.150	6.68	-1.20	-18.40	-6.09	-0.64	3.51	10.48

Source: Salomon Smith Barney.

Monitoring Performance with PCs

There is one complaint that we frequently hear about principal components. When developing the Independent Hedges and replicating portfolios above, we did so by measuring the dollar exposure to a one-standard-deviation move in each of the components. Unfortunately, investors and traders do not relate easily to this exposure. Is a $1,000 sensitivity large or small? When the curve changes, how can one use these exposures to estimate performance?

In contrast, duration and partial durations are easier for investors to interpret. With duration, a duration-dollar mismatch of $1,000 implies a return gain of $1,000 if yields decline by 1bp. And a five-year partial duration-dollar mismatch of $1,000 implies a return gain of $1,000 if five-year yields decline by 1bp (and yields at the other six partial duration maturities remain unchanged).

[16] We have also observed that if one allows too many (for example, all Treasuries) in the optimization universe, one risks obtaining non-intuitive solutions. Moreover, sometimes with a large optimization universe, highly leveraged solutions may result when a hedge solution includes both long and short positions.

The Independent Trades give us the means by which to provide similar interpretations of PC exposures. The change in yield of each of the hedge securities for a one-month one-standard-deviation move of each PC is shown in Figure 9. For each component, the magnitude of the one-standard deviation move can be captured by a single number — the change in the duration-dollar-weighted (DD-weighted) yield/spread of the Independent Trade, where the DD-weighted yield is calculated using the DD weights in Figure 4. Thus, a one-standard deviation move in PC No. 1 represents a 27bp increase in yields, in PC No.2 is a 17bp steepening of the curve (mainly between 2s and 30s), and in PC No.3 is a 3.5bp narrowing of the weighted 2s-10s-30s butterfly spread.

Therefore, the Treasury Index has a -$30 million exposure to a 27bp increase in the duration-dollar-weighted yield of securities in Trade No. 1, or a -$1.11 million exposure to a 1bp increase. Similarly, The Treasury Index has a $69,400 exposure to a 1bp widening of the duration dollar weighted yield spread of the 2s versus 10s and 30s slope trade. Moreover, the Index has a -$3,429 exposure to a 1bp narrowing of the 2s-10s-30s duration dollar weighted butterfly spread. By following these three duration-dollar-weighted yield/spreads — which represent the 3 PCs — investors should be able to estimate and understand their portfolios' performance.

Figure 9. Scenario Yield Changes for the Hedging Securities

	Level	Slope	Curvature
Two-Year	28.40 bp	-13.40 bp	1.00 bp
Ten-Year	28.70	0.50	-1.80
30-Year	24.80	4.60	1.70

Source: Salomon Smith Barney.

Summary of Hedging Results

Our results are encouraging. We have shown that the three principal components can be interpreted intuitively in terms of three independent trades. Trades involving these issues can capture the major sources of variation in the yield curve. Moreover, the Treasury Index can be well represented by a portfolio of only three issues, and yet the tracking errors appear to be reasonable. We end this section by raising several issues.

- The Treasury Index has virtually no optionality. Thus, our hedges provide reasonable matches to the convexity of our Index. Convexity obviously will be more of an issue with mortgage and corporate portfolios.
- The selection of the three issues to use for hedging/replication should depend on the application. For example, for the one- to five-year Treasury Index, 2s, 3s, and 5s will likely provide better hedges than 2s, 10s, and 30s. The general rule of thumb is to adequately distribute the three issues along the relevant portion of the yield curve.
- Individual security hedges likely will contain more tracking error than will the index as a whole, which is confirmed by initial work on hedging a long zero.

- While 2s/5s/25s provided less tracking error versus the Treasury Index than 2s/10s/25s, the latter may be more appropriate when a portfolio includes sizable weighting in STRIPS in the ten- to 20-year sector of the yield curve.

Practical Issues (or Frequently Asked Questions)

When using PCA for portfolio applications, a number of issues arise as to how stable the components are or how applicable the results are to different situations. We consider some of these questions below.

Do the PCA factors vary over time?

The PCs are derived from the covariance matrix of the weekly yield changes from 1989 to 1998. An important issue to explore is whether the same results would be obtained using data from a portion of that period. We found that data from different periods gave rise to qualitatively the same type of components: the first always plays the role of a level shift, while the second and the third play the role of a slope and curvature change, respectively. The first three PCs explained more than 99% of the variance, regardless of the data period.

However, there are some differences in the precise characterization of individual PCs. For instance, during the 1997–1998 period, the PCs of both the first and the second components have a much flatter shape than that in most previous periods. This suggests that, over this period, the curve has shifted more in a parallel fashion. Furthermore, the hump in the first component seems to have moved from about the three-year point out to longer maturities (about the seven-year point for 1997–1998). None of this is surprising, given the relative stability of the Fed and the yield curve over this period.

Nevertheless, when taken as a whole, the first three components are able to consistently explain the variation in the yield curve. We found that the first three PCs from any given sub-period can be replicated very closely by some linear combination of the PCs from the entire 1989–1998 period. Therefore, our first three PCs, when used together, form a very good proxy for movements in the yield curve over any sub-period from 1989 through today. The difference is in the magnitude of the variance of each PC. The hedging results we highlighted in the previous section reinforce our belief that the long-term PCs should be preferred in general, because they subsume the largest variety of monetary policies and economic conditions. PCs from a shorter time period may work better during that period (in-sample), but are likely to be inferior over the long run (out-of-sample).

Do the PCs depend on the data frequency?

Each PC remains fairly stable, regardless of the length of the interval over which yield changes are computed. However, the length of the interval affects the variance of the PCs. Not surprisingly, a comparison of weekly and monthly changes shows that the variance is proportional to the length of the period. For example, the weekly change PCs have one-fourth the variance of the monthly change PCs; the three-month yield change PCs have three times the variance of the monthly change PCs, and so on.

Are the PCs based on par and spot curves consistent?

The PCA results discussed thus far were based on movements in the par curve. Applying PCA to spot curves produces similar qualitative interpretations. Namely, the first three components represent level, slope, and curvature. Transforming the spot curve PCs to the par space, we found that the par curve movements implied by the

spot curve PCs are essentially the same as the original par curve PCs. Consequently, it does not matter which set of PCs we use to model the yield curve movements.

What happens if the first component is forced to be a parallel shift?

A portfolio constraint that a portfolio match the duration of a benchmark is the same as requiring immunization to a parallel shift of the curve. To study the effectiveness of this constraint, we repeated the PCA forcing the first PC to be a parallel shift.[17] As before, the second and third components represent slope and curvature components. Figure 10 shows the comparison of the variances explained by the two sets of PCs. The first three components in both sets of PCs explain the same percentage of the total variance. However, the parallel shift explains less of the total variance than the original first component because the magnitude of the hump is not present in the parallel shift. However, the lack of hump in the parallel shift is made up for by a more pronounced hump in the third PC than its counterpart in the original set of PCs. Any shape that can be produced by the parallel shift PCs can be replicated effectively by the original PCs, and vice versa. This indicates that if we hedge against a parallel shift (i.e., match durations) *and* the slope and curvature components, we will do just as well as we would if we hedged against the optimal PCs. Conversely, a hedge constructed using the original three PCs tends to produce a duration-neutral portfolio.[18]

Figure 10. Variance Explained by the Original and Parallel Shift PCs

	Cumulative Proportion of Variance		
	Component No.1	Component No. 2	Component No. 3
Original PCs	93.5 %	98.4 %	99.4 %
Parallel Shift PCs	92.3	97.3	99.4

Source: Salomon Smith Barney.

Should we use PCs computed from yield levels or yield changes?

This is a hotly debated issue. As a general rule, we believe that yield change PCs are more suitable for hedging and risk management purposes. This is typically the way we have been using PCA on the yield curve. In hedging and risk management, the emphasis is on minimizing the adverse effects of short-term movements in the yield curve. This is achieved by the constant rebalancing of one's portfolio. As a result, longer-term mean-reversion — which would involve yield levels — of the yield curve is less relevant for day-to-day management of portfolio risk.

Before we get into the discussion, let us first give a caveat: PCA does not recognize the time series aspect of its input data, be it yield levels or changes. In other words, given the data (for example, the weekly changes), we could scramble their order in time, do PCA on the scrambled data set, and would still obtain the same PCs. The implicit assumption in PCA is that the data are independent and identically distributed. This assumption would be violated, for example, if we were doing PCA on yield changes, and the changes depended on the yield levels. Of course, what we just described, i.e., changes depending on levels, is the gist of mean-reverting term-structure models, which have intuitive appeal. In contrast, the assumption of

[17] Actually, we applied PCA after first removing the parallel shift component from the weekly yield changes.

[18] This is because the parallel shift is closely approximated by a linear combination of the original three components.

independent and identically distributed changes is consistent with a random walk model, and, in particular, could give rise to negative yields. Therefore, strictly speaking, simple-minded PCA on yield changes cannot be consistent with a mean-reverting framework, in which changes would exhibit positive correlation (i.e., a trend) if levels were either at too high or too a low value in their ranges. However, this trending behavior in changes may not be statistically significant in the intermediate range of level values, especially when the yield curve is close to its "typical" shape. Because levels are more likely to take values around this typical shape than around extreme curve shapes, change data will likely be dominated by samples with little or no observable serial correlation. This reasoning provides some comfort about the assumption of independence in the change data.

Another issue is how long a time period to use to calculate the PCs. In general, it is better to use a very long time period. That way, periods of different monetary policies and economic environments are included, and any period-specific idiosyncrasies of the data have less of an influence on the calculated PCs (or, at least any quirky behavior is balanced by something opposite that is present in other periods of the data). As a result, analysis of long-term data should indeed give rise to reliable PC estimates. This is true for both level and change PCs.

The foregoing discussion highlights the shortcomings of PCA based solely on changes or levels. Either approach would leave some questions unanswered. It seems that a more promising solution could be found by somehow combining the two extremes. That solution, we believe, probably starts with modeling the yield curve as a stochastic process, very much in the spirit of term-structure models. On the other hand, we should reiterate that, with judicious use, rudimentary PCA has proven to be a very powerful tool for portfolio management (especially in Treasuries).

Does the fourth PC ever become important?

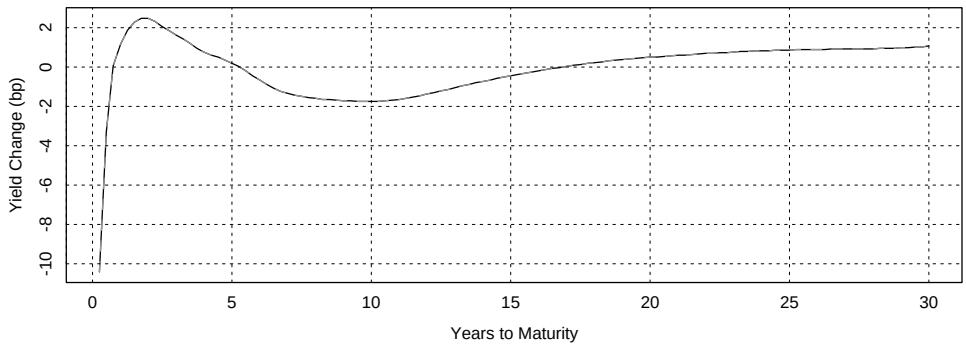
Despite the fact that the first three PCs explain more than 99% of the variance in the yield curve movements *in the long run*, there may be periods when the higher order PCs may have more explanatory power than the implied sub-1% level. However, from our experience, such episodes are rather rare and short-lived. So, hedging should not be based on the higher order PCs in general, and definitely not at the expense of a hedge against the first three PCs. Moreover, the fourth PC (or any of the higher-order PCs) is not as stable as the first three PCs. In other words, the reshaping implied by the fourth PC (not just the volatility of it) changes drastically depending on the time period used (see our discussion on the stability of PCs above). Any hedge based on such a fickle statistic will not be reliable. That said, a portfolio that is perfectly hedged against the first three PCs will have its primary curve exposure because of the fourth PC. But again, since the variance of that PC is very low, the residual P&L volatility due to this exposure will likely be small. For reference, Figure 11 shows the fourth PC. It implies a richening of the five- to 15-year sector of the curve relative to the two-year sector and the long end. An interesting interpretation of the exposure to the fourth PC is provided by viewing the exposure as relative value in a portfolio. For example, if recent movements in the yield curve observed in the market imply an extreme realization for the fourth PC (several standard deviations), then one could expect a reversion of this behavior, such that the long-term standard deviation and the zero-mean property are maintained. This means that there is a possible relative-value

opportunity in the market. From that perspective, an exposure to the fourth PC could be a measure of the possible P&L if

Are the yield curve PCs suitable for investors with exposure to the front end of the curve only?

a reversion (an opposite realization of that PC) does indeed occur, assuming that the first three PCs are hedged.

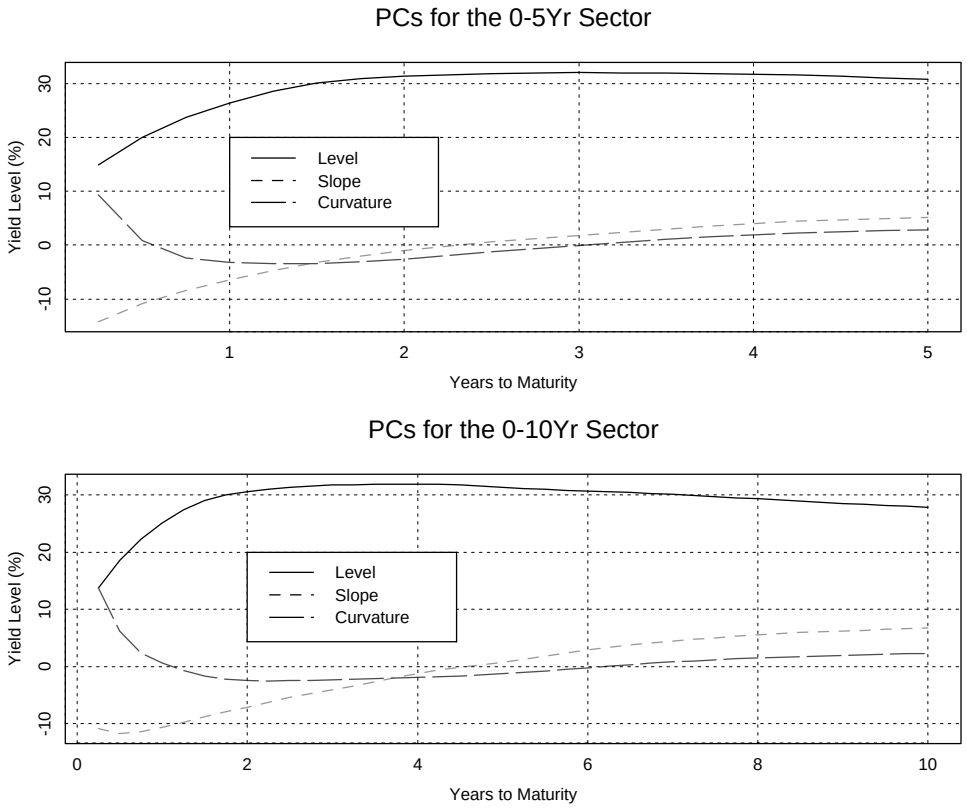
Figure 11. The Fourth Principal Component for a One-Month Horizon Computed from Weekly Yield Changes, Jan 89-Feb 98



Source: Salomon Smith Barney.

The PCs we have described so far are designed to model the movements of the entire yield curve. For those investors whose assets and/or liabilities are concentrated in the short end, PCs that are custom-made for that end may be a more accurate model. Figure 12 shows the PCs for the 0- to five-year and 0- to ten-year maturity ranges. Despite the differences in their values and where they intersect the 0bp line, these PCs possess certain common characteristics with their full-maturity-range brethren: the first PC is still a level shift with no zero-crossing, the second a slope change, and the third, a curvature change. The first three PCs explain 99.5% and 99.4% of the variance in the 0- to five-year and 0- to ten-year range, respectively. What is interesting is that one can closely replicate either set of short-end PCs by some combination of the 0- to 30-year PCs. However, the weights used in these combinations represent several standard deviation movements for the 0- to 30-year PCs, which are not very likely. Turning this argument around, a one-standard-deviation move in the 0- to 30-year PCs could give rise to larger movements in the short end than warranted by an analysis of that sector alone. As a result, the exposures for a short end portfolio obtained from the 0- to 30-year

Figure 12. The First Three Principal Components for a One-Month Horizon Computed from Weekly Yield Changes in the 0- to Five-Year and 0- to Ten-Year Sectors, Jan 89-Feb 98



Source: Salomon Smith Barney.

PCs may not adequately represent the risks in the portfolio. Our conclusion is that a short-end portfolio is better managed with the short-end PCs.

Part II — Principal Components for Structuring Butterfly Trades

- **Butterfly trades allow one to identify and trade rich/cheap sectors of the curve. PCA provides a method by which to structure curve-neutral butterfly trades that isolate the relative-value opportunity from any market- and slope-directional bias.**
- **The Salomon Smith Barney Butterfly Model, available through SSB Direct, provides a powerful platform from which to look for relative value using butterfly trades. One can quickly analyze trading signals and hedging properties from many perspectives using different weighting methods and several types of data, such as Treasury model data, futures CTDs, coupon Treasuries, and STRIPS.**
- **After extensive historical simulations, we conclude that (i) the Butterfly Model using PCA on yield levels as the weighting method is preferable to PCA on yield changes; and (ii) the analysis should be based on a long-enough (three or four years) window of data.**

We are frequently asked about butterfly trades or specific sectors of the curve. To provide a framework for answering these questions, we have developed a novel Butterfly Model. This model is available through an interactive screen on our Salomon Smith Barney Direct Internet site, under US Governments/Research Models. The model, which can be applied to any three-legged trade, offers a new analytical approach to evaluating butterflies. The new application can be used to:

- Evaluate which sectors of the curve — coupon Treasuries or STRIPS — are rich and cheap.
- Analyze the market and slope directionality of any butterfly weighting.
- Identify the appropriate weights for a butterfly trade under a variety of methods.

The Butterfly Model is easy to use and incorporates several operations into one, producing a concise summary of a particular butterfly trade. We are making our constant maturity Treasury model and STRIPS curves available for use in the butterfly model.

What Makes a Good Butterfly Weighting Scheme?

Some investors do butterflies to take on market exposure, some desire a market-neutral relative- value trade. They will use different weightings.

Investors undertake butterfly trades for two main reasons: (1) to take on market direction or slope exposure within a cash and/or duration constraint, or (2) to undertake a curvature or relative-value trade.

1. In the first case, the duration-dollar (DD) neutral investor, for example, would sell bullets to buy barbells when expecting the curve to flatten, and would buy bullets when expecting the economy to slow and the Fed to ease.

2 In the second scenario, the relative-value investor desires to take advantage of the relative cheapness or richness of one security (center) compared to two securities (wings) around it, without taking a market directional view. The expectation is that the center security will come back in line with the wings. This investor will want to make the assessment of value independent of changes in market direction or curve slope, and will want to know how to weight the trade to eliminate market or slope biases.

Duration-dollar-neutral butterflies have market exposure. Our Butterfly Model assesses "how much."

The two most common butterfly weightings are: (1) a 50-50 weighting (putting 50% of the duration-dollars in the short wing and 50% in the long wing), and (2) a cash-neutral and duration-neutral weighting. While these weightings match the duration-dollars of the buy-side and the sell-side, they ignore the fact that the three yields may have different volatilities and imperfect correlations. Therefore, these common weighting schemes typically produce residual market-level and slope exposure. Our Butterfly Model can be used to evaluate the magnitude of these exposures.

PCA has advantages over common statistical methods for weighting market-neutral butterflies.

In order to structure market-neutral butterflies for pure relative-value plays, investors look toward statistical methods, such as regression analysis[19] or volatility weighting.[20] These techniques get us part of the way there, but can have some shortcomings. To construct a butterfly-weighting scheme that produces a pure market-neutral curvature play, we recommend using principal components analysis (PCA) on the yields of the three legs of the trade. Principal component analysis[21] is a general statistical method used to analyze the variation in multivariate series. Using both the historical volatility and correlation structure among the three yields, PCA will identify the weightings to best immunize a butterfly trade to both market-level and slope factors[22] — the first two principal components — while leaving the exposure to curvature (or relative value), which is the residual variation among the three yields. Furthermore, because of a fundamental property of PCA, the PCA-weighted butterfly spread is not correlated with market level and slope (the first and second components). This characteristic is the key to constructing curve-neutral butterfly trades.

PCA weights are generally not duration-dollar-neutral, but they are market-neutral.

Because PCA is not based on the premise of a parallel shift, the duration-dollar weights of the wings can, and typically do, add up to something other than 100%.[23] With a traditional duration-dollar matched butterfly trade, buying the center is typically a trade that is long the market in reality. By selling a little bit more than 100% of the duration-dollars on the buy-side, the market directionality is removed,

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[19] In this application, for example, ordinary regression analysis suffers from the ambiguity involved in specifying the independent and dependent variables. That is, the betas depend on what is regressed against what. For example, X regressed on Y and Y regressed on X may result in two betas that are not the inverse of each other (the ratio of the two betas is the ratio of the variances of X and Y). This discrepancy, in turn, gives rise to different duration-dollar weights, depending on how the regressions are run.

[20] Volatility weighting recognizes the term structure of volatility across the curve, but assumes perfect correlation between points on the curve. In effect, it is a one-factor approach to modeling the curve.

[21] See also our writings on PCA for the entire yield curve, which appeared in the August 1, 1997, issue of *Bond Market Roundup: Strategy*.

[22] No *a priori* definition of market level or slope is required. The technique finds the best way to hedge systematic variation among the three yields. The PCA weighting produces the butterfly spread with minimum variation. The residual variation can be thought of as the relative value or curvature play.

[23] Similarly, the weightings do not necessarily produce cash-neutral trades.

but at the expense of the duration-dollar match. We believe that because PCA provides a more realistic yield curve model than a parallel-shift model, investors should not necessarily insist on matching duration-dollars.[24]

Use the PCA framework to identify cheap sectors, even if executing with other weights.

Whether or not the PCA weights are used in executing a trade, we believe that they are a useful framework for evaluating the relative value of a sector. In summary:

- As many traditional butterfly spreads are market-directional, the PCA analysis allows one to evaluate when a sector of the curve has cheapened or richened *beyond* that prescribed by recent yield movements. If a sector looks cheap based on PCA weights (versus the wings) *and* a traditional butterfly has a bullish bias, the butterfly trade is likely to be a good way to go long the market (with an added relative-value kicker).
- The PCA analysis is a good way of identifying rich and cheap sectors of the yield curve, in which the relative valuation is independent of market direction. Portfolios of the cheap sectors can then be structured into curve-neutral portfolios, or into portfolios that have desired market exposures.

Overview of the Butterfly Model Input and Output — What to Look For

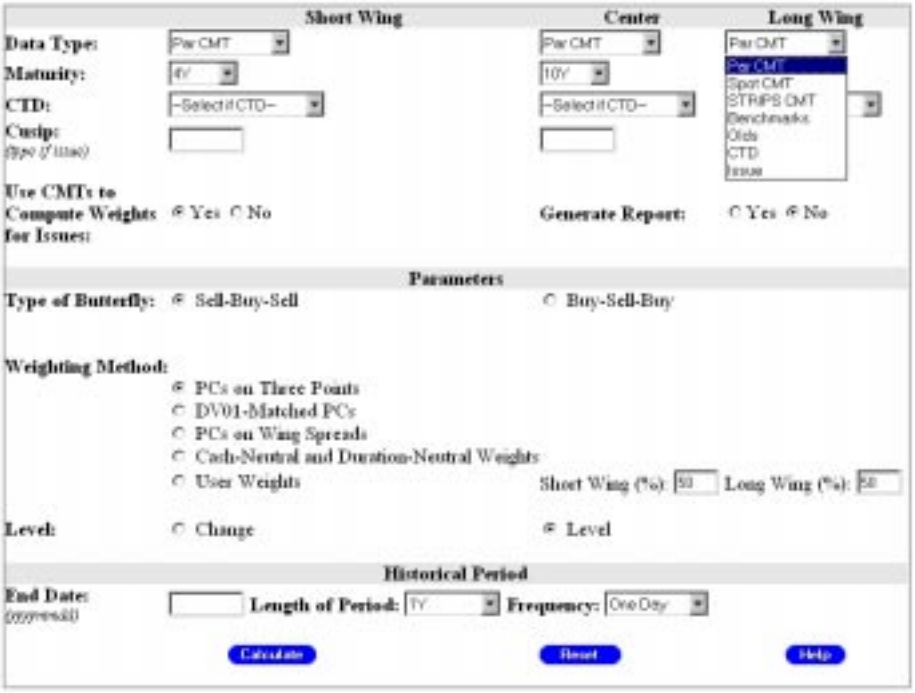
We recommend using the CMT Treasury Model or CMT STRIPS yields for the analysis.

We offer the user a variety of historical yield series to use in the analysis. Because issues roll down the curve, over time shortening their durations and maturities, we prefer to analyze constant maturity yield series. We have made our constant maturity (CMT) Treasury Model curve and CMT STRIPS yield series available. However, users can also choose to analyze the yield series of specific notes, bonds, and STRIPS, if so desired[25] (see Figure 13). In our example, we use CMT Treasury Model par curve data to analyze a 4s-10s-27s butterfly, in which we buy 10s. Our weighting method for the example will be PCA applied to the yields of the three points on the CMT par curve.

[24] However, keeping in mind those investors who might be constrained to be duration-dollar matched, in the butterfly model we also provide an option to run PCA with a slight modification to get weights that ensure duration-dollar matching.

[25] Other alternatives include the yield series of rolling benchmarks, rolling old benchmarks, and the issues that are currently the cheapest-to-deliver into the Treasury futures contracts.

Figure 13. Various Data Models Available on the Butterfly Model Input Screen



Source: Salomon Smith Barney.

The main output is a page of eight graphs.

The main output of the Butterfly Model is a page of eight graphs for assessing the relative value and historical performance of the trade (see Figure 14). The recommended duration-dollar weighting for the trade is also reported on the page of graphs. The first, third, and fourth rows have pairs of graphs: the left, a scatter plot, and the right, the time series of the residuals (distance from point to the fitted line) of the scatter plot. The current value of the residual, beta, correlation, and the percentile of the residual over the history are reported below the graphs. The second row shows the time series of the butterfly spread[26] obtained using the calculated weights, as well as the time series of market level and slope. While there is much information on this page of eight graphs, we suggest focusing on the following:

Focus on the following items:

1 **"Quick" Relative Value:** The first row gives a quick read on relative value by scattering the spread of the long wing[27] versus the spread of the short wing. This is equivalent to looking for value in a 50-50 weighted butterfly trade. Although not perfect, this is commonly used by market participants. If the current point is above the fitted line, as in Figure 13, it suggests that the middle of the butterfly is cheap.

2 **Recommended Weights:** The recommended DD-weights for the trade, calculated according to the method specified by the user, are listed below the left

[26] Our definition of butterfly spread differs from market convention and is more directly related to the P&L potential in the trade; see the Appendix for a detailed discussion of this issue.

[27] Because we are long the center in this example, the long wing spread is defined as 10s minus 27s, and the short wing spread as 10s minus 4s, so that a point below the fit line indicates richness of the buy-side security.

graph in row 2. The graph shows the duration-dollar weighted yield spread[28] of the butterfly. In our example, 36 duration dollars in 4s and 67 duration dollars in 27s would be sold for every 100 duration dollars of 10s bought.

3 **Correlation with Market Level:** The left graph in Row 3 evaluates the correlation of the trade with market level. A near-zero correlation results in a horizontal fit line through the points. Look for a very high (or low) residual percentile (printed below the right graph), which indicates a good time to buy (or sell) the center.[29] The value of the current residual is the deviation of the current spread (in bp) from the average and, therefore, is an indication of the expected return potential from the trade.

4 **Correlation with Slope:** The graphs in Row 4 evaluate the correlation of the trade with curve slope. Again look for very high or very low percentiles.

5 **Mean Reversion:** It is important to evaluate the pattern of the butterfly spread and the residuals for mean reversion. We prefer butterflies with spreads (or residuals) that periodically cycle in a band — i.e., high values should follow low values, and vice versa, in a somewhat cyclical fashion. If the spreads (or residuals) are not cycling but trending, we would suggest further analysis of the trade and the factors potentially driving the relative value. Perhaps a longer historical time frame would be appropriate.

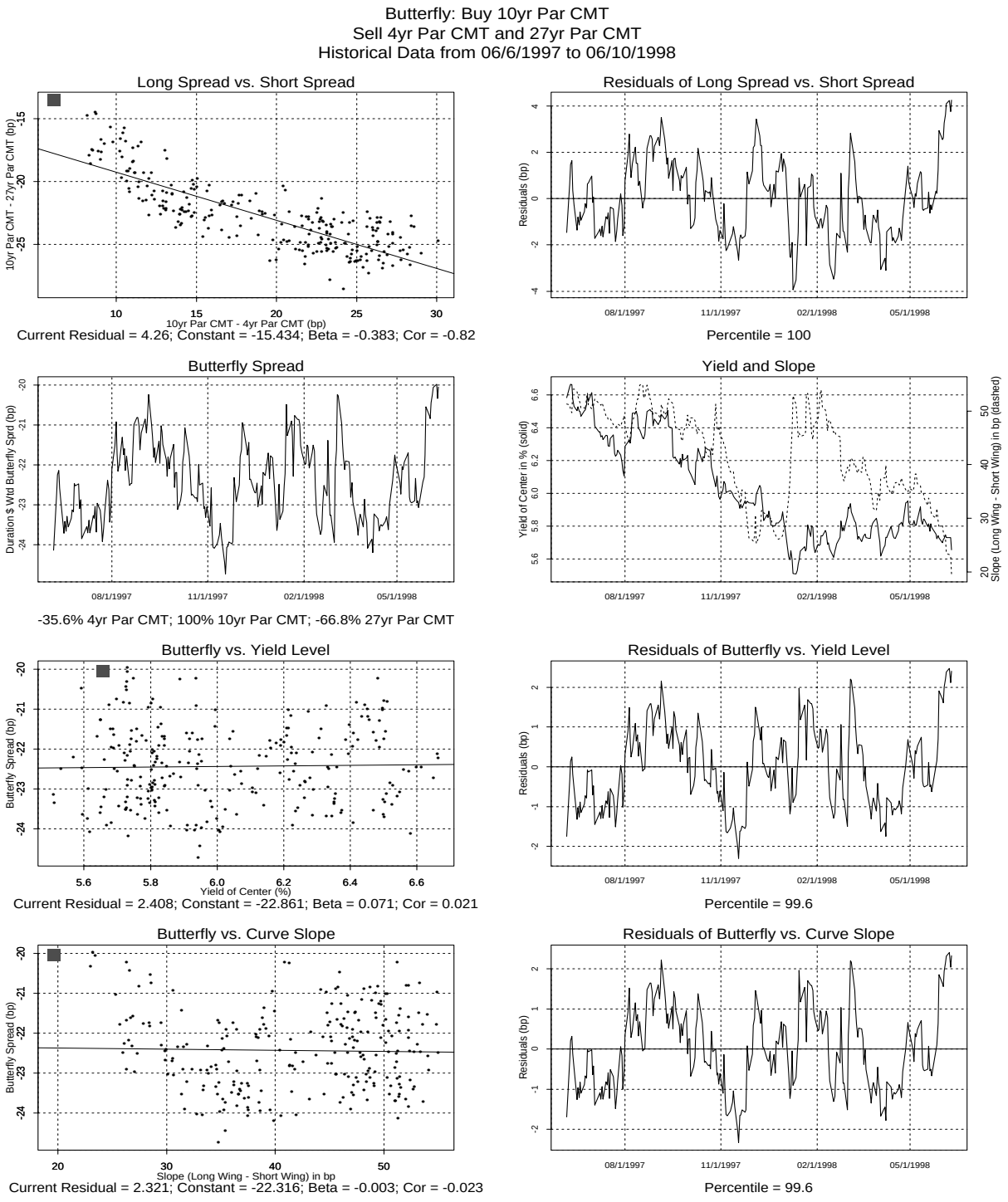
The model also produces a trade report with details.

If requested, a trade report (see Figure 15) can be generated that details the execution of the trade with specific securities. The report includes the par, market value, and duration-dollar weights for the trade. The report will include cash if needed to match market values.

[28] In our example, the DD-weighted yield spread = (yield of center) - 36% * (yield of short wing) - 67% * (yield of long wing). This definition is different from the traditional butterfly spread — defined as twice the yield of the center minus the sum of the yields of the wings. Note that, even in the case of a 50-50 weighted butterfly trade, the DD-weighted spread will be one half the traditional butterfly spread.

[29] We prefer this percentile to that in (1) above as an indication of relative value, as it removes market-directional biases.

Figure 14. The Results of the Butterfly Model Run for 4s-10s-27s Trade



Butterfly weightings computed via principal components on yield levels of
4yr Par CMT, 10yr Par CMT, and 27yr Par CMT

Source: Smith Barney Inc./Salomon Brothers Inc.

Figure 15. Trade Report for the Following Trade: Buy 5.625s of May 08, Sell 6.5s of May 02 and 6.875s of Aug 25

Price Date: **06/10/1998**
Settle Date: **06/12/1998**

Type	Cusip	Coupon	Maturity	Duration	Yield	Price	Par ($M)	Mkt ($M)	Dur$ ($M)	Dur$ (%)[a]
US	9128272W	6.500	05/31/2002	3.464	5.593	103.184	-7,539	-7,779	-26,948	-36
US	9128274F	5.625	05/15/2008	7.510	5.513	100.844	10,000	10,084	75,735	100
US	912810EV	6.875	08/15/2025	12.915	5.795	114.687	-3,413	-3,914	-50,555	-67
Net								-1,609	-1,768	-2

[a] Computation method used: PCs on Three Points

Note: The duration weights were calculated using the following CMT data for the issues:

4yr Par CMT

10yr Par CMT

27yr Par CMT

Source: Salomon Smith Barney.

For more info, push the Help button.

For a more in-depth discussion of the analytics behind our Butterfly Model, or for a more complete User's Guide to the interactive options, please see the help file on Salomon Smith Barney Direct, which can be accessed via the input screen.

Levels or Changes? A Dilemma Revisited

After extensive studies, we have concluded that it is better to use PCs on levels for butterfly trades, rather than on changes, to generate the trading signals and to compute the trade weights. Furthermore, although probably not as critical as the choice between levels and changes, one should use a very long historical data period to calculate the PCs.

When getting into a butterfly trade, we are primarily interested in taking advantage of an anomaly in the yield levels of three securities. That suggests using levels as the starting point. But here is the puzzle: once we decide that the yield levels present a trading opportunity, we will likely want to set up a curve-neutral trade. In other words, we will look to put on a position that is hedged against yield level shifts and slope changes. This would suggest using the statistics of yield changes to weight the trade. Unfortunately, trade weights obtained from levels generally do not match those obtained from changes. The level weights may not hedge against incremental changes in level and slope, but can hedge against the aggregate changes in these quantities. By construction, the PC-weighted butterfly spread structured from levels will not be correlated with yield level and slope. This means that as the butterfly spread sweeps its range between the two extremes, any *short-term* correlation between the butterfly spread changes, level shifts, and slope changes is washed out. Therefore, the best way to put on a butterfly trade is through level PCs, as they provide not only the trading signal, but also the best long-term curve-neutral position.

The other important question is related to the length of the time period used to calculate the weights. Should one use two years of history, or three, four, or even longer? As we stated at the beginning of this section, we have concluded that using a very long time period appears to be best. On the other hand, there could be some legitimate concerns with that approach. Are the old data still relevant to the market today? Should we not put more weight on more recent data, as they probably better capture any effects of the current monetary policy? It may appear that recent data are more appropriate for what is happening in the market today. However, by using data

from a short period that reflects a specific monetary policy regime and economic conditions, one implicitly assumes the continuation of the same environment going forward. The yield statistics obtained that way may be completely off in the case of a policy change. In contrast, a longer time period will likely include cycles of tightening as well as easing, strong economic growth as well as slowdown. Therefore, yield statistics — and the weights derived from them — have a better chance of signaling profitable trade opportunities and providing better curve-neutrality properties under a wide variety of conditions. Put another way, using long-term weights that capture different economic environments allows one to avoid relying on the current economic environment to continue.

Now, let us turn to some examples. We start with a comparison of the PCA weights calculated using levels and changes on two trades: a 2s-5s-10s trade, and a 5s-7s-10s trade. For the x-year history, we compute the weights for these trades daily using data from rolling windows of varying lengths (two-year, three-year, four-year, and five-year). Then for each experiment, we calculate the volatility of these weights and the excess duration (sum of two weights minus the weight of the center) over the x-year period.[30] Figure 16 shows the results.[31]

Figure 16. Comparison of the Volatility of Butterfly Weights and Excess Duration Calculated Using Levels and Changes Over Rolling Two-Year, Three-Year and Four-Year Windows, Jan 93-Dec 99

	5s-7s-10s											
	Volatility of Weight of 5s				Volatility of Weight of 10s				Volatility of Excess Duration			
Rolling Window	2-Yr.	3-Yr.	4-Yr.	5-Yr.	2-Yr.	3-Yr.	4-Yr.	5-Yr.	2-Yr.	3-Yr.	4-Yr.	5-Yr.
Levels	5.5 %	5.1 %	2.7 %	1.5 %	5.2 %	5.2 %	3.1 %	2.1 %	0.5 %	0.5 %	0.6 %	0.7 %
Changes	2.4	1.5	2.5	2.7	2.1	1.3	2.8	3.4	1.0	0.6	0.6	0.7

	2s-5s-10s											
	Volatility of Weight of 2s				Volatility of Weight of 10s				Volatility of Excess Duration			
Rolling Window	2-Yr.	3-Yr.	4-Yr.	5-Yr.	2-Yr.	3-Yr.	4-Yr.	5-Yr.	2-Yr.	3-Yr.	4-Yr.	5-Yr.
Levels	8.4 %	6.9 %	2.0 %	0.9 %	7.1 %	6.3 %	2.1 %	1.1 %	2.0 %	1.1 %	0.9 %	1.1 %
Changes	3.0	2.2	1.0	0.9	3.4	2.9	1.7	1.4	1.7	1.6	1.2	0.7

Source: Salomon Smith Barney.

Overall, these examples imply that changes give more stable weights for the two wings than levels do. This is particularly evident in the 2s-5s-10s trade. In terms of excess duration, neither method is clearly more stable. In all cases, there is typically more stability of both weights and excess duration when a longer window of historical data is used to compute the weights. Although not shown in the figure, it is

[30] In calculating the volatilities, we only use the data prior to the beginning of the crisis during the fall of 1998. This is done to ensure that the statistics are not disproportionately affected by the extreme market movements during and after the crisis. However, the results do not change significantly, and the conclusions remain the same, even when the crisis period is included in the calculations.

[31] These volatilities are in absolute terms: if the mean weight on 2s in 2s-5s-10s is, say, 35%, and the volatility of the weight is 10%, then the one-standard-deviation band is between 25% and 45%. On another note, in general, higher volatility in one weight is accompanied by higher volatility in the other weight also. This is because usually the weights simply get shifted from one wing to the other. In other words, when one weight changes, the other changes by about the same amount, but in the opposite direction. For example, if 2s have a weight of 35% and 10s a weight of 70%, then when the weight on 2s goes up to 40%, the weight on 10s goes down to about 65%. Surely this is just a rough approximation, because the sum of the two weights does not have to remain the same and the fluctuation in the sum is captured in the volatility of the excess duration.

also interesting that the long-term mean of each weight appears to remain the same, independent of the length of time used to calculate the weights. This suggests that even though a shorter window may result in higher volatility of weights, *the average* weights are consistent with weights obtained from longer periods.

Although stability of the weights is desirable, it is not necessarily the goal. A better gauge of the effectiveness of the butterfly weights is whether the trades recommended and structured by the model generate profits. To test the efficacy of the model, we ran historical simulations. We used the change in the butterfly spread as a measure of the P&L. For example, if we put on a long trade (long the center, short the wings) and the butterfly spread decreases by 1bp, then we make a profit of 1bp times the duration dollars in the center of the trade. Conversely, a short trade (short the center, long the wings) makes a 1bp profit if the spread increases by 1bp.

Here is how the simulations work. The trading signals are obtained from the percentiles of the (PC-weighted) butterfly spread over a rolling window of a specified length (two-year, 3-year, etc.). Whenever the model indicates that the butterfly spread is at a low (high) percentile, called the start level, we put on a short (long) trade. Then when the spread reverts back to a higher (lower) level, called the exit level, we take off the trade. The reversion to the exit level is meaningful only if it does not take too long to happen. So, we put a limit on the maximum number of days we would be willing to keep the trade on (the holding period); if the reversion has not happened by then, we take off the trade, regardless of the P&L situation. That way, we get out of any trade within a reasonable amount of time, and then we are free to put on other trades if opportunities arise.

Before running the simulations to compare levels and changes, we have tested several different combinations of start and exit levels, and holding periods to optimize the choice of these parameters. The results are shown in Figure 17. The start and exit levels are in the leftmost column. For example, the first set of start/exit values is 3/10, indicating that we put on a short trade when the spread dips below 3% (or a long trade, when the spread exceeds 97%), and take it off when the spread reverts to the 10% level (or the 90% level). We tried three different holding periods of 60, 120, and 365 days.

The first observation is that waiting up to 365 days to end a trade results in poor performance. The second observation is that the higher P&Ls and Sharpe ratios are concentrated in the upper part of the table, with start/exit percentiles of 3/10, 3/30, and perhaps 10/30. This leads us to believe that the P&L on these trades is realized at the tail ends of the spread range, and that there is no advantage in looking for trading signals in the intermediate percentiles. In choosing between using 3% or 10% as the level to put on a trade, we note that while the 10% level gives rise to more trades, the mean P&Ls are not higher, and the Sharpe ratios are decidedly lower than with 3%. Therefore, we favor the 3% level to start the trades. As for the choice between using 10% or 30% as the exit level, we prefer 30%, due to the higher average P&Ls; besides, the Sharpe ratios for 30%, while lower than those for 10%, are still fairly good.

Figure 17. Performance of 2s-5s-10s Trade Structured Using Yield Levels for Various Start/Exit Levels and Maximum Holding Periods of 60, 120, and 365 Days, Jan 93-Dec 99

Start/Exit	No. of Trades			Probability of Success			Percentage of Trades Taken Off at Day Limit			Mean P & L			Volatility of P & L			Sharpe Ratio		
Levels	60	120	365	60	120	365	60	120	365	60	120	365	60	120	365	60	120	365
3/10																		
2-Year	20	17	16	85%	88%	94%	30%	18%	6%	2.1bp	1.9bp	1.4bp	2.6bp	3.6bp	7.2bp	0.81	0.53	0.19
3-Year	17	14	12	82	86	100	24	21	0	2.4	2.4	3.5	1.9	2.1	2.0	1.26	1.14	1.75
4-Year	14	12	12	86	100	100	14	0	0	3.1	3.6	3.6	3.5	2.7	2.7	0.89	1.33	1.33
3/30																		
2-Year	18	14	8	83%	79%	62%	50%	43%	38%	2.2bp	1.4bp	-0.5bp	3.0bp	4.3bp	10.5bp	0.73	0.33	-0.05
3-Year	16	12	9	81	75	89	62	42	11	2.7	2.9	4.8	2.7	3.5	3.3	1.00	0.83	1.45
4-Year	13	9	6	85	89	83	62	33	17	4.2	5.6	6.6	5.0	5.4	6.5	0.84	1.04	1.02
10/30																		
2-Year	29	21	12	72%	76%	75%	48%	43%	25%	1.2bp	1.3bp	0.0bp	2.8bp	3.8bp	8.6bp	0.43	0.34	0.00
3-Year	25	19	14	68	74	93	52	32	14	1.5	2.3	3.3	2.9	3.3	2.6	0.52	0.70	1.27
4-Year	23	15	11	83	80	91	52	40	18	2.9	3.7	4.8	3.9	4.4	4.7	0.74	0.84	1.02
10/50																		
2-Year	28	17	10	64%	65%	70%	68%	59%	30%	1.2bp	1.0bp	-0.1bp	3.5bp	4.8bp	9.6bp	0.34	0.21	-0.01
3-Year	24	15	11	62	67	82	62	53	36	2.2	2.2	3.2	3.8	4.6	4.5	0.58	0.48	0.71
4-Year	21	15	10	71	73	90	71	53	20	3.0	3.6	5.9	5.1	5.9	6.0	0.59	0.61	0.98
30/50																		
2-Year	38	25	12	58%	60%	67%	63%	44%	33%	0.4bp	0.6bp	-1.5bp	3.4bp	5.2bp	10.6bp	0.12	0.12	-0.14
3-Year	38	24	15	55	62	73	66	50	27	1.4	2.1	2.8	3.6	4.4	5.0	0.39	0.48	0.56
4-Year	39	27	19	62	70	79	59	37	21	1.9	3.1	3.8	4.3	5.0	4.9	0.44	0.62	0.78

Source: Salomon Smith Barney.

Next, we ran the simulations to compare levels versus changes. We used the rolling PC weights indicated by the yield levels and yield changes over two-year, three-year, and four-year periods. We also computed the PC weights over an 11-year period — again both for levels and changes — and then went back and applied those fixed weights over rolling two-year windows during the same 11-year period. This is different from the other simulations, in that it is an in-sample test. Finally, we computed butterfly weights, such that we are hedged to the first two PCs of the entire yield curve.[32] Figure 18 shows the performance of the trades for each one of the different ways of weighting.

[32] For example, for the 2s-5s-10s trade, the yield changes implied by the first two PCs (in bp) are 28.4, 30.5, 28.7 and -13.5, -7.1, 0.5. A weighting vector that hedges out the first two PCs is orthogonal to these two 3x1 vectors, and is given by the vector product of the two vectors. Then we normalize the resulting orthogonal vector such that its weight on 5s is 100%. The final result is -54.5% weight on 2s, 100% on 5s, and -52.3% on 10s.

Figure 18. Comparison of Performance of Butterfly Trades Using Levels and Changes Over Rolling Two-Year, Three-Year and Four-Year Windows, Over a Fixed Ten-Year Window, and from the PCs of the Entire Yield Curve, Jan 93-Dec 99

2s-5s-10s

	No. of Trades		Probability of Success		Percentage of Trades Taken Off At Day Limit		Mean P & L		Volatility of P & L		Sharpe Ratio	
	60	120	60	120	60	120	60	120	60	120	60	120
2-Year												
Level	18	14	83%	79%	50%	43%	2.2bp	1.4bp	3.0bp	4.3bp	0.73	0.33
Change	14	11	57	64	86	55	1.0	1.3	3.7	5.5	0.27	0.24
3-Year												
Level	16	12	81%	75%	62%	42%	2.7bp	2.9bp	2.7bp	3.5bp	1.00	0.83
Change	9	8	78	75	44	25	2.8	3.4	4.2	3.2	0.67	1.06
4-Year												
Level	13	9	85%	89%	62%	33%	4.2bp	5.6bp	5.0bp	5.4bp	0.84	1.04
Change	8	7	100	100	50	43	5.7	5.9	3.5	5.1	1.63	1.16
11-Year												
Level	11	10	82%	80%	36%	30%	2.8bp	3.6bp	3.8bp	2.9bp	0.74	1.24
Change	18	14	78	86	44	36	1.7	1.7	2.9	3.9	0.59	0.44
Full Curve PCs	14	10	64	80	71	50	1.4	1.8	4.4	6.7	0.32	0.27

5s7-10s

	No. of Trades		Probability of Success		Effect of Day Limit		Mean P & L		Volatility of P & L		Sharpe Ratio	
	60	120	60	120	60	120	60	120	60	120	60	120
2-Year												
Level	20	16	80%	94%	30%	19%	1.2bp	1.4bp	1.2bp	1.1bp	1.00	1.27
Change	14	10	57	70	79	50	0.9	1.3	2.0	2.3	0.45	0.57
3-Year												
Level	13	10	77%	70%	54%	30%	1.2bp	1.2bp	1.5bp	1.9bp	0.80	0.63
Change	11	9	82	89	64	33	1.6	2.3	2.0	1.8	0.80	1.28
4-Year												
Level	14	9	79%	56%	71%	67%	1.2bp	0.6bp	1.3bp	1.7bp	0.92	0.35
Change	9	7	67	71	89	43	0.6	1.7	1.4	1.9	0.43	0.89
11-Year												
Level	16	11	75	73	69	55	1.0	1.1	1.7	1.4	0.59	0.79
Change	14	10	71	70	64	50	0.8	1.0	1.4	1.4	0.57	0.71
Full Curve PC	16	11	69	73	69	64	0.7	0.9	1.6	1.0	0.44	0.90

Source: Salomon Smith Barney.

First of all, whether structured using levels or changes, the trades make money, as indicated by the high probability of success (percentage of trades that make money), and the positive overall average P&L. Furthermore, for a wide range of the trade parameters, the Sharpe ratios are very good (i.e., above 0.5), which is very comforting.

The comparative performance results can be summarized as follows: (1) in a vast majority of cases, levels give rise to a larger number of trades; (2) success probabilities between levels and changes under identical conditions are comparable,

and typically high (upward of 70%) especially if a long window is used; (3) in five out of eight cases for 2s-5s-10s and six out of eight cases for 5s-7s-10s, levels have same or better Sharpe ratios than changes; (4) longer time windows appear to boost average P&L for 2s-5s-10s, but not so for 5s-7s-10s, but there is no advantage in using the 11-year history (instead of say, the three-year history), even though the results of the 11-year case are in-sample; (5) waiting longer (120 days instead of 60) for the profit potential to be realized may increase average P&L, but at the expense of higher volatility, so Sharpe ratios do not necessarily improve; (6) the trades that use weights from PCs of the entire yield curve have the worst performance, indicating that specific trades are better managed using PCs that are tailor-made for those trades, rather than by the generic ones for the curve.

As a result of these observations, the choice between levels and changes is still a tough one, but we opt for levels for two reasons: the higher number of trades and the (generally) higher Sharpe ratios. Because the average P&L does not improve drastically by increasing the holding period for trades, and the Sharpe ratios are acceptable across the board, we prefer a shorter holding period (60 days in this study) so that we have more trades. Finally, regarding the length of the window, we prefer longer windows (three years or longer), due to the higher average P&Ls, as Sharpe ratios are fairly high using levels and a 60-day maximum holding period no matter what the window length. However, we should caution against using a very long period (11 years, in our study), because there does not seem to be an advantage in terms of P&L and the number of trades.

An interesting question is whether levels and changes give the same trading signals, i.e., whether the timing of the trades is identical. The answer is no: in a majority of cases, the two approaches give different trading signals. The average P&L of the 2s-5s-10s trade is a respectable 4bp or 5bp if the trades are structured with levels in the optimal way. However, the 5-7s-10s trade has an average P&L of about 1bp with levels, no matter what the length of the time window used. So, one could naturally ask whether this is a worthwhile trade, especially taking into account the bid-offer spread. We should emphasize that our historical simulations are based on CMT yield data derived from our Treasury model. Therefore, what our results establish is a simple proof of concept: the model makes money. In reality, one would set up these trades with actual issues, and ideally sell issues that are rich (to the model) and buy those that are cheap. That could easily add another few basis points to each trade. Furthermore, our CMT data come from closing prices. But during a trading day, there can be large fluctuations in the butterfly spread — because of an economic announcement, for instance — that can create very good trading opportunities. Because investors quickly take advantage of such extreme relative value opportunities, these opportunities disappear just as quickly, usually before the day is over. Finally, P&L can be further boosted by using futures instead of cash. Therefore, our results should be viewed as a conservative estimate of the profitability of trades structured using the butterfly model.

To conclude this section on levels versus changes, we would like to remind our readers that PCA ignores the fact that yields are time series. Although we believe that level PCs give a more than adequate answer to timing and setting up a butterfly

trade, it is conceivable that the results can be improved further by using a model that exploits the time series properties of yields. At the very least, such a model would put to rest the question of whether to use levels or changes by implicitly combining the two in a consistent framework.

Lessons From the Crisis of Fall 1998

Before we conclude, we would like to reiterate the importance of using a long-enough time window (three or four years). With a shorter window, periods of unusual yield curve movements or single monetary policy regimes may affect the statistics disproportionately. A very important example of this is related to the crisis during the fall of 1998. In particular, because of the demand for the liquidity of the on-the-run issues, the off-the-run sectors cheapened relative to the on-the-runs and have not yet fully recovered. As a result, the shape of the curve underwent a dramatic change, with a more dominant hump in the 21-year sector than was observed previously. The 9s-21s-28s trade demonstrates this effect. Using a two-year history prior to the crisis, PCA would give weights of -5% for the nine-year and -96.7% for the 28-year (the minus signs indicate that these securities are sold against a purchase of 21s). However, the same analysis at the beginning of November 1999 gives +34.3% for the nine-year and -149.3% for the 28-year, i.e., one has to *buy* the nine-year and sell more of the 28-year to hedge level and slope risk! This is no longer a butterfly trade. What happened is that with the onset of the crisis, the curve became more humped, so the hump (or curvature) started to explain a higher percentage of the curve shape. Since the first two PCs picked this up, the third PC was distorted to such an extent that it could not be interpreted as curvature any more. One can still construct a butterfly of 9s-21s-28s, by excluding the crisis period from the analysis (e.g. getting the PCA weights from the pre-crisis or post-crisis period). Using nine months of data in the post-crisis period, the weights are -8.3% and -94.9% — much more sensible than the weights from the two-year period including the crisis. Another possibility is to use cash-neutral and duration-neutral weights, but this is likely to be an inferior choice, as it does not make use of the statistics of the yields. The best alternative is to use a longer time window: using a four-year window, the weights do not exhibit those peculiar swings: using pre-crisis data, the weights are -6.6% and -95.4%, compared with -2.8% and -95.8% from the period including the crisis.

Appendix A – Computation of the Principal Components

Let $y_1, y_2, \ldots, y_K$ be K different (scalar) time series. The value of the nth sample, or observation, of the kth time series is denoted by $y_k(n)$. In our applications, the y_Ks are yield levels or yield changes at different maturity points. In PCA, for the entire yield curve, we have y_1 through y_{120} (K = 120) that represent the time series of 120 CMT points spanning the maturity range up to 30 years with quarter-year increments. For butterfly applications, we have three time series (K=3) that represent the yields (or changes) of the securities in the three legs of the trade.

Let **y**(n) denote the value of the Kx1 vector of the K time series $y_1, y_2, \ldots, y_K$, at time n, i.e.

$$\mathbf{y}(n) = \begin{bmatrix} y_1(n) \\ y_2(n) \\ \vdots \\ y_K(n) \end{bmatrix}$$

Being a vector in a K-dimensional space, **y**(n) can be written in terms of a given (fixed) set of K orthonormal[33] Kx1 vectors $\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_K}$

$$\mathbf{y}(n) = c_1(n)\mathbf{a}_1 + c_2(n)\mathbf{a}_2 + \cdots + c_K(n)\mathbf{a}_K$$

for some scalar coefficients $c_1(n), c_2(n), \ldots, c_K(n)$. Because the $\mathbf{a_k}$s are orthonormal, for each $\mathbf{y}(n)$ there is a distinct and unique set of coefficients $c_1(n), c_2(n), \ldots, c_K(n)$ that satisfy the equation above. These coefficients are given by

$$c_k(n) = \mathbf{y}(n)^T \mathbf{a}_k, \text{ for } k = 1, \cdots, K$$

Clearly, the value of the coefficients $c_1(n), c_2(n), \ldots, c_K(n)$ for a given **y**(n) depends on the choice of $\mathbf{a_k}$.

Given N observations of the Kx1 vector **y,** denoted by $\mathbf{y}(1), \mathbf{y}(2), \ldots, \mathbf{y}(N)$, one can calculate a coefficient $c_k(n)$ for each observation $\mathbf{y}(n)$. This gives, in effect, N observations of the coefficient c_k. Given these N observations, it is a simple matter to calculate the variance of each coefficient c_k.

The objective in principal components analysis is to find a set of orthonormal **a**'s such that the first coefficient c_1 has the largest variance possible for any choice of $\mathbf{a_1}$, the second coefficient c_2 has the second-largest variance, and so on. The problem of finding $\mathbf{a_1}$ that gives the highest variance reduces to the following form:

maximize $\mathbf{a}_1^T S \mathbf{a}_1$ subject to $\mathbf{a}_1^T \mathbf{a}_1 = 1$,

where S is the covariance matrix of **y**. The solution to this constrained optimization problem is given by the eigenvector of S associated with the largest eigenvalue. In fact, the highest variance thus found *is* the largest eigenvalue. Similarly, $\mathbf{a_2}$ is the solution to the problem

[33] An orthonormal set of vectors $\mathbf{a_1}, \mathbf{a_2}, \ldots, \mathbf{a_K}$ satisfies two properties: i) the vectors have unity vector norm, and ii) the vectors are orthogonal to each other.

maximize $\mathbf{a}_2^T S\mathbf{a}_2$ subject to $\mathbf{a}_2^T\mathbf{a}_2 = 1$ and $\mathbf{a}_2^T\mathbf{a}_1 = 0$

and is given by the eigenvector of S associated with the second-largest eigenvalue. Again, the second largest variance turns out to be equal to the second-largest eigenvalue. The remaining orthonormal vectors $\mathbf{a}_3, \ldots, \mathbf{a}_K$ are computed in a similar way. The vectors $\mathbf{a}_1, \ldots, \mathbf{a}_K$ are called *loadings* and the coefficients $c_1, c_2, \ldots, c_K$ are called *principal components* (note that in this report, we refer to the vectors loosely as principal components to reduce the statistical jargon).

The importance of principal components is that a large part of the variance in the vector **y** can be explained by a relatively small number of principal components. The fundamental relationship is the following:

$$\sum_{k=1}^{K} Var(y_k) = \sum_{k=1}^{K} Var(c_k)$$

In other words, the total variance in the vector **y** is equal to the sum of the variances of the principal components, i.e., each principal component explains part of the total variance. In our yield curve PCA application, the first three principal components (the ones associated with the largest eigenvalues) explain 99.4% of the total variance. In contrast, in the butterfly application, there are three yield time series (K=3), so the first three PCs are the only PCs and as such they represent all of the variance.

Appendix B – Methodologies for Computing Portfolio Hedges Using Principal Components[34]

Computing the Independent Trades

We suggest two methods for computing the Independent Trades that represent the three PCs. Fundamental to both methods is the selection of the hedge securities and the calculation of scenario dollar returns in the Yield Book™. We have concluded that the best hedges occur when the three securities are “well-spaced” along the curve and are representative of the portfolio being hedged. Because we will be using these independent trades to hedge the Treasury Index, we have selected the **on-the-run two, ten, and 30-year** Treasury issues. We suggest using the one-month PC scenarios ("pc1mo" scenario file) in the Yield Book™. This scenario file contains six scenarios, two for each PC: for example *comp1+* and *comp1-* are the respective positive and negative realizations of the first component. These scenarios are scaled to the appropriate one-month one-standard-deviation for each component. Calculate returns using a zero-day horizon.

1. Analytic solution for computing the Independent Trades. This approach utilizes analytic and algebraic techniques to construct the three Independent Trades. First, we need the sensitivities of each hedge security to each PC. Using the Yield Book™, calculate scenario dollar returns for the one-month horizon PC scenarios. The sensitivity of an issue to the first component equals the dollar return under *comp1+* minus the dollar return under *comp1-*, divided by two. Similar calculations are done for the other components. The next step is to solve

[34] Parts of this appendix originally appeared in the March 6, 1998, issue of *Bond Market Roundup: Strategy, Salomon Smith Barney*.

a system of linear equations to determine each Independent Trade. The coefficients in this system are the PC sensitivities, and the unknowns are the trade weights for each security. The system of equations is defined in such a way that the solution set of trade weights produces a portfolio with exposure to only one component at a time.[35] It is worth mentioning that such a system is well-defined and has a unique solution, since the PCs are independent by construction and we are using three hedging securities to hedge out the three exposures.

2 Optimization solution for computing Independent Trades. Using the optimization feature of the Yield Book™ on chapter 4, pg. 2, the weights for each Independent Trade can be easily obtained. Bring in the three hedge securities onto both the buy and sell sides.36 Once the scenario returns have been calculated, perform three optimizations — one for each Independent Trade, as follows. Enter the objective and constraints of the optimization: Constrain the dollar return differences of the positive PC scenarios (one will be positive and two zero) and "maxmin" over the negative PC scenarios. For example, to calculate the Independent Trade to hedge the first component (a level shift of the yield curve), constrain the dollar return difference of the first scenario, comp1+, to some amount, say $1 million, set the dollar-return difference of comp2+ and comp3+ equal to zero and "maxmin" the dollar-return differences of comp1-, comp2-, and comp3-.37 Repeat for comp2+ (a change in slope) and comp3+ (a change in curvature).

[35] For each i=1, 2, 3, solve: W(1)*S(1,k)+ W(2)*S(2,k)+ W(3)*S(3,k)=E(i), where E(i)=1 if i=k and 0 otherwise, where S are the sensitivities, W are the security weights and E the exposures to each component.

[36] Since we do not know *a priori* whether the solution will be short or long each issue.

[37] Because of convexity, constraining comp1+ to be 1,000 and comp1- to be 1,000 and the other return differences to be zero will not produce a feasible solution.

Figure 19. Independent Hedge Trades for $1 Million Exposure to a One-Month One-Standard-Deviation Move in Each Component (Market Value in $ Thousands), 27 Feb 98

	Analytic Solution			Yield Book Optimization Solution		
	Independent Trade 1 -- Level					
	EDUR$	Weight	Market Value	EDUR$	Weight	Market Value
2-Year	-$5,686	15.42%	-$30,537	-$5,768	15.38%	-$30,976
10-Year	-16,735	45.40	-22,033	-17,021	45.38	-22,409
30-Year	-14,442	39.18	-10,600	-14,715	39.24	-10,800
	-$36,863	**100.00%**		**-$37,503**	**100.00%**	
	Independent Trade 2 -- Slope					
	EDUR$	Weight		EDUR$	Weight	
2-Year	$57,281	100.00%	$307,633	$57,051	100.00%	$306,395
10-Year	-14,480	-25.28	-19,064	-15,010	-26.31	-19,762
30-Year	-49,055	-85.64	-36,004	-49,584	-86.91	-36,392
	-6,254	**-10.92%**		**-$7,543**	**-13.22%**	
	Independent Trade 3 -- Curvature					
	EDUR$	Weight		EDUR$	Weight	
2-Year	-$71,838	-25.14%	-$385,813	-$71,854	-25.25%	-$385,899
10-Year	285,769	100.00	376,235	284,535	100.00	374,611
30-Year	-248,715	-87.03	-182,544	-249,858	-87.81	-183,383
	-34,784	**-12.17%**		**-$37,177**	**-13.07%**	

Source: Salomon Smith Barney.

In Figure 19, we show the size of each trade that will provide a $1 million profit (exposure) to a one-standard-deviation move (over a one-month period) in each component for both methods. These trades each have exposure to only one PC and provide an intuitive meaning of the nature of the three PCs. These two procedures return nearly identical solutions.

Computing Portfolio Hedges

For computing hedges of a portfolio, we suggest four possible methods:

1 **Calculate the hedge from the Independent Trades** that have been calculated via one of the methods above. Here, the Yield Book™ can be used to the dollar sensitivities of the portfolio, or index, to the three components through scenario analysis with our "pc1mo" scenario file. Since each Independent Trade creates exposure to only one component, each trade can be sized to offset the portfolio's dollar sensitivity to the corresponding component.[38] These three sized trades can then be aggregated to determine the amounts of the three securities that comprise the hedge. Figure 20 displays the hedge for the Treasury Index resulting from the scaling of the Independent Trades created through the optimization method (method 2) detailed above.

[38] Simply scale the market values in each Independent Trade by the portfolio's dollar exposure to a component divided by $1 million.

Figure 20. Independent Hedge Trades for the Treasury Index for a One-Month One-Standard-Deviation Move in Each Component ($ in Millions), 27 Feb 98

	Independent Trade 1 Level		Independent Trade 2 Slope		Independent Trade 3 Curvature		Total		
	Market Value	Eff Duration $	Market Value	Eff Duration $	Market Value	Eff Duration $	Market Value	Eff Duration $	% of Duration $ in Index
2-Year	$926,996	$172.6	$362,466	$67.5	-$4,631	-$0.9	$1,284,831	$239.2	21.5%
10-Year	670,608	509.4	-23,379	-17.8	4,495	3.4	651,724	495.0	44.4
30-Year	323,197	440.4	-43,052	-58.7	-2,201	-3.0	277,944	378.7	34.0
Total	**$1,920,802**	**$1,122.3**	**$296,035**	**-$8.9**	**-$2,336**	**-$0.4**	**$2,214,500**	**$1,112.9**	**99.9%**
Index Dollar Return	**-$29,926**		**$1,183**		**$12**				

Source: Salomon Smith Barney.

2 **A direct analytic solution** is similar to the analytic solution we proposed to calculate Independent Trades. The first step is to determine the dollar sensitivities to each PC for each hedge security and the portfolio. Next, we proceed to solve a linear system of three equations with three unknowns. The variables are the trade weights that would match the dollar exposures of the hedge with the dollar returns of the portfolio. The coefficients of the system are the computed sensitivities of the hedge. Therefore, the solution will be a hedge that matches the portfolio. Again, the system is well defined and has a unique solution.

3 **Using optimization over the PC scenarios,** a hedge solution can be determined without identifying the Independent Trades. Bring the portfolio to be hedged onto the sell side and the three hedge securities onto the buy side. Calculate scenario dollar returns for the one-month PC scenarios and, through optimization, "maxmin" the dollar-return difference over all six scenarios. (Dollar return is used because market value and duration dollar need not match, although in many applications, they will be close).

4 **Optimization over the combo scenarios** can also be used. The procedure is as described above in (3). However, this time we choose the one-month combo scenarios ("cmb1mo" scenario file) that were created from the PCs. (This is the set of scenarios containing *bearflat, bearsteep, bearintrm*, etc.) Again, "maxmin" the dollar-return difference over the six combo scenarios.

In Figure 21, we show the three-security portfolios that were calculated to replicate the Treasury Index through these four methods. For hedging the Treasury Index, we found that all the methods produce similar results. All methods described above lead to very similar hedges with a small tracking error. We would not expect solutions to be exactly the same, because the four methodologies described above are qualitatively different. In particular, the first method uses only three components to the hedge, while methods 2 and 3 use six (three positive and three negative) components to calculate a hedge. Since positive and negative components are a mere reflection of each other and the convexity effect is negligible (especially for the PC2 and PC3), the solutions are very close. The fourth solution uses six combo scenarios to the hedge. The combo scenarios are derived by taking linear combinations of individual PCs. From the construction of combination scenarios, we would expect the solutions (2), (3), and (4) to be very close, but there is no reason why they should match exactly. For ease of implementation, we currently recommend using optimization over the PC scenarios (method 3).

Figure 21. Hedge Trades for the Treasury Index for a One-Month One-Standard-Deviation Move in Each Component ($ in Millions), 27 Feb 98

	Independent Trades		Analytic Solution		PC Optimization		Combo Optimization	
	Market Val.	%EDUR$	Market Val.	%EDUR$	Market Val.	%EDUR$	Market Val.	%EDUR$
2-Year	$1,284,831	21.50%	$1,284,118	21.49%	$1,284,991	21.50%	$1,281,059	21.44%
10-Year	651,724	44.48	651,749	44.49	651,437	44.46	653,621	44.62
30-Year	277,944	34.03	277,863	34.02	278,115	34.05	277,230	33.95
	$2,214,500	**100.00%**	**$2,213,730**	**100.00%**	**$2,214,543**	**100.01%**	**$2,211,910**	**100.01%**

Source: Salomon Smith Barney.

Hedging a Portfolio Versus an Index

Taking our analysis one step further, if you have a portfolio that is benchmarked against the Treasury Index and you would like to see if it is well hedged for movements in the Treasury yield curve, the following can be done:

1. Scale the market value of the index to equal the market value of your portfolio. Compute the hedges for the index and your portfolio, separately, under one of the methods detailed above. You will then have two portfolios of three securities apiece that can be compared to determine what trades, if any, need to be put on to match the exposures of the index.
2. Bring your portfolio onto the buy side and the index onto the sell side. Again, scale the market value of the index to that of your portfolio. Bring in the selected hedge securities onto both the buy and sell sides and use method 3 or 4 (optimization over PC or combo scenarios). Any additional securities on the buy side and/or the sell side represent trades that will neutralize the exposures of the portfolio versus the Index for those scenarios.

Appendix C – What Is the "Right" Butterfly Spread to Assess Performance in a Butterfly Trade?[39]

Our butterfly model allows investors to identify rich and cheap sectors of the curve and to set up curve-neutral butterfly trades to take advantage of these relative-value opportunities. The optimal allocation of the duration-dollars in the trade is calculated automatically by the model, based on the objectives and constraints specified by the investor. Discussions with users of our butterfly model often raise the issue that many investors follow 50-50 weighted butterfly spreads, which may not accurately track the performance of these trades. We provide an overview of the spreads that ultimately are related to the P&L of the trade.

The value in a butterfly trade lies in the relative cheapness or richness of a bullet against a barbell. An opportunity exists when the shape defined by the yields of the three securities is more humped than observed in historical data: In this case, an investor would want to own the bullet and sell the barbell. Conversely, when the shape has much less of a kink than in the past (or a more pronounced negative kink), it means that the bullet is rich relative to the barbell, and one would sell the bullet and buy the barbell.

The return on the butterfly trade is due to two factors:[40]

[39] The material in this appendix originally appeared in the March 12, 1999 issue of *Bond Market Roundup: Strategy*.

[40] This analysis is linear and ignores the effect of convexity.

The yield on the composite position, and

The change in yields of the three securities.

The conventional spread assumes an equal allocation of the duration-dollars between the two wings.

The short-term relative-value investor expects to benefit primarily from the second factor, as a result of a change in the "spread" between the bullet and the barbell. Here the spread is understood to be a measure of the curvature of the three yields. The market convention is to quote this spread on a 50-50 basis.[41]

$$\textit{Market Convention Spread} = 2y_{bullet} - \left(y_{short\,wing} + y_{long\,wing}\right)$$

Therefore, when this spread decreases, being long the bullet and short the barbell should result in a profit. However, unless the *duration-dollars* are allocated equally between the two wings of the barbell (50% in the short wing and 50% in the long wing), the change in this spread is only loosely tied to the actual profit in the trade.

Confounding the issue of the P&L of a butterfly trade is the convention of quoting the yield pickup on the same 50-50 basis using the above equation. As mentioned above, the yield pickup on the composite position determines one component of the return. However, calculating this component of the return in the trade using the quoted 50-50 yield pickup could again be misleading, unless the total *market value* in the barbell is allocated equally between the two wings.

To see how the yield and the butterfly spread should be calculated for an arbitrary butterfly trade, one can start by reviewing the terms in the return expression for the trade. First, the change in the value[42] P of a bond can be approximated by:

$$dP \cong Pydt - PDdy.$$

In this equation, dP is the change in value, D is the duration of the bond, y is the yield, and dy and dt denote the change in yield and time, respectively. The product PD is the duration-dollars in the bond position. The first term on the right is the yield return and the second term is the return due to a change in yield.

The P&L in a butterfly trade is the difference between the change in value of the buy side and the change in value of the sell side. For simplicity, assume that the trade is cash-neutral, i.e., that the market values of the buy and sell sides are the same.[43] Denoting this market value by P and the duration-dollars on any leg of the trade by DD, the P&L of a long-bullet/short-barbell trade is given by:

---

[41] $y_{short\ wing}$ and $y_{long\ wing}$ denote the yields on the short-maturity and long-maturity securities of the barbell, respectively.

[42] This analysis is linear and ignores the effect of convexity.

[43] If there is a cash mismatch in the trade, the return on the cash should also be included in the return calculations. This cash component is invested (or borrowed) using short-term instruments. Because of the small duration on these instruments, the return on cash is primarily owing to yield. When the trade is kept on for a short time, this yield return is small but should not be ignored.

$$dP_{bullet} - dP_{barbell} = P\left(y_{bullet} - \left(\frac{P_{short\ wing}}{P} y_{short\ wing} + \frac{P_{long\ wing}}{P} y_{long\ wing}\right)\right) dt$$
$$- DD_{bullet}\left(dy_{bullet} - \left(\frac{DD_{short\ wing}}{DD_{bullet}} dy_{short\ wing} + \frac{DD_{long\ wing}}{DD_{bullet}} dy_{long\ wing}\right)\right)$$

The return due to yield is related to the market-value-weighted yield...

From this equation, it is clear that the yield pickup in the trade is given by the following equation:

$$Yield\ Pickup = y_{bullet} - \left(\frac{P_{short\ wing}}{P} y_{short\ wing} + \frac{P_{long\ wing}}{P} y_{long\ wing}\right)$$

This is simply the *market-value-weighted* yield of the position. Therefore, the market convention represents twice the yield pickup for a 50-50 market-weighted butterfly. In many butterfly trades, the total market value is far from being equally distributed between the two wings, making the market convention a rather inaccurate way of measuring the actual yield pickup.

The return owing to the yield changes is given by the product of the duration-dollars of the bullet and the following quantity:

$$dy_{bullet} - \left(\frac{DD_{short\ wing}}{DD_{bullet}} dy_{short\ wing} + \frac{DD_{long\ wing}}{DD_{bullet}} dy_{long\ wing}\right)$$

while the return due to yield changes is related to the duration-dollar-weighted yield spread.

The last expression is the change in a duration-dollar-weighted yield spread, defined as follows:

$$Butterfly\ Spread = y_{bullet} - \left(\frac{DD_{short\ wing}}{DD_{bullet}} y_{short\ wing} + \frac{DD_{long\ wing}}{DD_{bullet}} y_{long\ wing}\right)$$

From this viewpoint, it is easy to see that the market convention corresponds to twice the butterfly spread for a 50-50 duration-dollar-weighted butterfly. When the actual duration-dollar allocation differs from 50% on each wing, the market convention spread gives only a distorted view of what is happening with the trade. In contrast, the duration-dollar-weighted butterfly spread:

- **Gives the correct measure of the cheapness or richness in the actual trade, and**
- **Provides a simple way to measure the sensitivity to a change in yields.**[44]

As an example, consider a hypothetical butterfly of three securities with yields of 4%, 5.25%, and 5.5%. Suppose that the market values in the trade are $65 million, $90 million, and $25 million, and the durations are 1.538, 3.333, and 8. The duration-dollars are $10,000, $30,000, and $20,000. Then the market value weights

[44] For example, for a total of $100,000 duration-dollars in the bullet, a 1bp change in the duration-dollar-weighted butterfly spread translates to a $100,000 change in the value of the position.

are 65/90 = 72.2% and 25/90 = 27.8% for the short wing and long wing, respectively. The duration-dollar weights, on the other hand, are 10/30 = 33.3% and 20/30 = 66.7%. Therefore, the yield pickup is 83.3bp (= 5.25 - (0.722*4 + 0.278*5.5) = 0.833%) and the duration-dollar-weighted butterfly spread is 25bp (= 5.25 - (0.333*4 + 0.667*5.5) = 0.25%). However, one half of the 50-50 spread is 50bp.

The change in the 50-50 spread may not be correlated with the changes in the duration-dollar-weighted butterfly spread.

Not only are all these spreads different levels, providing different indications of any yield pickup in the trade, but as yields change over time the changes in these spreads are not necessarily correlated. For example, suppose that over a one-week period, the short wing cheapens by 3bp and the long wing richens by 3bp, while the bullet's yield stays the same. This represents a flattening of the curve from the short wing to the long wing. In this case, the 50-50 spread is unchanged, but the duration-dollar-weighted spread widens by 1bp (= 0 - (0.333*0.03 + 0.667*(-0.03)) = 0.01%). Now consider the opposite case, in which the curve steepens from the short wing to the long wing: the short wing richens by 3bp, the long wing cheapens by 3bp, and the bullet's yield stays unchanged. The 50-50 spread is still 0bp. However, this time the duration-dollar-weighted spread tightens by 1bp (= 0 - (0.333*(-0.03) + 0.667*0.03) = -0.01%).

When the trade is not market-value-neutral, the return on cash should be included in the P&L of the trade.

We have assumed in this discussion that the trade that is executed is both market-value- and duration-dollar-neutral. However, this is not always the case and, if it is not, further complications and confusions can arise. When the duration dollars on the buy and sell sides are equal but cash is needed to make the market values equal, the duration-dollar-weighted spread of the three securities will be correlated with performance. However, the market-value-weighted yield used in evaluating the return due to yield pickup must also include the cash return. This is obviously more important the longer the holding period of the trade. Failure to include the cash return could make a winning trade look like a losing trade.

When using our butterfly model to compute weights based on three principal components, trades that are duration-dollar-neutral rarely result. In this case, the formula for the butterfly spread above still holds. However, the weights for the short and long wings will not add to one, as in the duration-dollar-neutral trade.[45]

[45] While this adds no complication to the application of that formula, it will for investors who compute butterfly spreads by weighting the short spread (yield bullet - yield short wing) and the long spread (yield bullet - long wing) by the weights in the long and short wings. This will no longer result in the same values.

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