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The Restructuring Clause in Credit Default Swap Contracts

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The new 2003 ISDA Credit Derivatives Definitions allow for four different clauses for handling restructurings as credit events that trigger default swaps: Old Restructuring, Modified-Restructuring, Modified-Modified-Restructuring and No Restructuring. We describe a model to price these different contracts that explicitly addresses the valuation of the protection buyer's cheapest-to-deliver option. As introduction, we first describe the contents of the restructuring clause and then discuss three known cases where the restructuring clause was triggered. In all three cases, the firms were forced to restructure to survive the refinancing of significant amounts of maturing bank debt.

1. INTRODUCTION

In the current standard credit default swap contract linked to a corporate (non-sovereign) reference credit, there are three credit events that can trigger the payment of protection. They are bankruptcy, failure to pay, and restructuring. Two years ago the mechanism for settling a default swap following a credit event was the same for all of these types of credit events. If a credit event occurred, protection buyers with physically settled contracts would settle them by delivering a face value amount of deliverable obligations in return for the face value amount paid in cash. The market standard was to allow the delivery of obligations with a maximum maturity of 30 years.

Today we have four different types of default swaps differentiated by their handling of the settlement of the default swap following a restructuring credit event: the old restructuring clause (Old-R), the deletion of restructuring as a credit event (No-R), the (American) modified restructuring (Mod-R), and the proposed[1] (European) modified-modified restructuring (Mod-Mod-R). The catalyst for this change was the restructuring of the US insurer Conseco, Inc. in September 2000.

The public announcement that Conseco had reached an agreement with a bank consortium to extend maturing bank loans for 15 months, was used by Conseco default swap protection holders as evidence that a credit event had occurred. The default swap holders used the broad 1999 ISDA definitions on deliverable obligations (Old-R) and settled the contracts with longer maturity deep discount bonds, trading in the 65-80 range. The event caused considerable controversy as the default swap sellers argued that protection buyers had purchased protection to reduce their credit exposure to Conseco's short-term loans and that the restructuring had not impaired these short maturity loans – they argued that this was a "technical default". Banks who had bought protection to hedge these loan exposures, and who were party to the restructuring, were able to take advantage of the delivery option by purchasing and delivering these cheaper long-dated bonds, thereby receiving a windfall gain, at the expense of protection sellers. It could be argued that this event highlighted a weakness in the credit derivative documentation. However, the aim of permitting a broad range of deliverables in a credit default swap is not to create a cheapest-to-deliver option but to enable

[1] *This new mechanism has been recently proposed and is expected to be adopted by the European market on 6 May 2003.*

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a standard contract that can be used to hedge the credit risk in a wide range of *pari passu* assets issued by a reference entity. While these assets should trade at the same price following a bankruptcy or failure to pay, this is not the case for a restructuring event. For this reason, restructuring needs to be treated as a separate case.

This was achieved in the US through the adoption of the Mod-R contract in May 2001. This standard was not adopted within Europe, where bank regulators were not ready to accept as a hedge a contract which severely constrained the basket of deliverables. After long consultations with market participants, ISDA introduced the Mod-Mod-R contract in March 2003, which gives the protection buyer a larger set of deliverable obligations than specified in the Mod-R contract.

The ISDA documentation specifies the conditions that must be satisfied for an agreement between a company and its lenders to qualify as a restructuring credit event that triggers default swaps. To estimate the likelihood that a particular company will restructure, it is necessary to understand the specific composition and terms of the company's debt. In the next section, we discuss the contents of the restructuring clause options incorporated into the new 2003 ISDA definitions. We explain the restrictions on deliverable obligations that must be taken into account when determining which obligation is the cheapest to deliver to settle the default swap.

In the third section we discuss three cases where the restructuring clause has been triggered, and a fourth case where a restructuring was a possibility. These cases indicate that if a company restructures it is likely to find itself in a situation where its credit has deteriorated, it is cash constrained, and it is having problems rolling over maturing bank debt. In the restructuring, the banks agree to extend the maturity of the debt or otherwise modify its terms to prevent forcing the company to seek protection in bankruptcy. Once a restructuring agreement has been reached, the likelihood that the company will default in the short term is diminished, and as such a restructuring is good-news for holders of short-term debt.

Finally, in section 4, we present a simple model to price differences in the restructuring clause. We modify the Jarrow-Turnbull credit pricing framework to explicitly incorporate a curve of restructuring probabilities as well as a curve of default probabilities after a restructuring.

We conclude in section 5.

2. THE RESTRUCTURING CLAUSE

We start by defining what qualifies as a restructuring, then we describe the different types of contracts.

2.1 What qualifies as a restructuring?

According to the 2003 ISDA definitions, a debt obligation is considered restructured if there is:

1) interest rate reduction,

2) reduction in principal or premium,

3) postponement or deferral (maturity extension),

4) change in the priority ranking of payments, or

5) change in currency or composition of payment of principal or interest.

The restructured obligation must be held by more than three unaffiliated holders and at least two-thirds of the holders must have consented to the restructuring.

It is also important to note that *the occurrence of, agreement to or announcement of any of the five triggers in circumstances where such trigger does not directly or indirectly result from a deterioration in the creditworthiness or financial condition of the reference entity shall not constitute a restructuring.*

2.2 Old-R

Before ISDA published the *restructuring supplement* to the 1999 definitions on 11 May 2001, default swaps traded with restructuring included as a credit event and with a straight 30-year maturity limitation on deliverable obligations. As discussed in the introduction, the Conseco case revealed the problems with this combination. Today, in the US, Old-R default swap trades are rare and confined to unwindings of existing positions. In Asia and Europe Old-R contracts are still the standard.

2.3 No-R

The No-R clause eliminates restructuring as a credit event and is a simple solution to the problems identified by the Conseco case. No-R has especially been advocated by JPMorganChase and a number of insurance companies.

2.4 Mod-R

The 11 May 2001 ISDA restructuring supplement introduced a number of changes. In particular, the documentation allowed parties to specify *restructuring maturity limitation applicable*. This is the Mod-R clause. It states that if a restructuring is the only credit event specified in the credit event notice, then the swap may only be settled with delivery of an obligation that matures before the *restructuring maturity limitation date* (RMLD).

To determine RMLD it is necessary to first find the following three dates:

1) Let LMD be the latest (possibly extended) maturity date of any restructured obligation.

2) Let RD be the restructuring date, ie, the date the restructuring is legally effective.

3) Let STD be the scheduled termination date on the default swap.

Now let M = min{LMD, RD + 30 months}. If M < STD then RMLD = STD. If M > STD + 30 months then RMLD = STD + 30 months. Otherwise, if STD ≤ M ≤ (STD + 30 months) then RMLD = M. This implies that RMLD is bounded between STD and STD + 30 months.

The formula for finding RMLD can seem confusing. An approximation that is often used is

$$\text{RMLD} = \max\{\text{STD}, \text{RD} + 30 \text{ months}\}$$

We examine the accuracy of this approximation in the example below.

Under Mod-R, an obligation must be *fully transferable*, meaning that it must be either a transferable bond or a loan that can be transferred to an *eligible transferee* without consent. In particular, a loan is not fully transferable if its transfer can be blocked by the borrower (the restructured firm). Under Mod-R, eligible transferee is broadly defined and does not take into account whether the transferee is prevented from owning the obligation for regulatory reasons.

2.5 Mod-Mod-R

The Mod-Mod-R clause was introduced with the new 2003 ISDA definitions and defines the restructuring maturity limitation date by the formula

$$\text{RMLD} = \max\{\text{STD}, \text{RD} + \text{A}\}$$

where STD and RD have the same definitions as section 2.4 and A = 60 months for restructured obligations and A = 30 months for all other obligations. Note that, unlike Mod-R, this definition is independent of the maturity of the restructured debt. It only depends upon the maturity date (STD) of the default swap and whether a restructured obligation is being delivered.

Under Mod-Mod-R, a restructured obligation must be *conditionally transferable*, which differs from fully transferable (as it applies to Mod-R) with respect to the definition of eligible transferee. The Mod-Mod-R definitions only require a loan to be transferable without consent to an entity engaged in the loan business.

In Figure 1 we compute the restructuring maturity limitation date for Mod-R and Mod-Mod-R contracts.

Figure 1. Comparison of restructuring maturity limitation dates for Mod-R and Mod-Mod-R contracts

				Mod-R		Mod-Mod-R	
Case	**RD**	**LMD**	**STD**	**Exact**	**Approx.**	**Restructured**	**Non-restructured**
1	0	24	12	24	30	60	30
2	0	36	12	30	30	60	30
3	0	72	12	30	30	60	30
4	0	24	48	48	48	60	48
5	0	72	48	48	48	60	48

All numbers are in months from today's date.

In Figure 1, under Mod-Mod-R for restructured obligations, the restructuring maturity limitation date is 60 months. For cases 3 and 5 the maturity of the restructured obligations is greater than 60 months and, hence, the constraint is binding. Under Mod-R, the situation is more complex, because the restructuring maturity limitation date depends on the maturity of the restructured obligations. In cases 2, 3 and 5 the constraint is binding.

2.6 Other provisions in the restructuring clause

1) The restructuring maturity limitation only applies when the credit event has been triggered by the protection buyer.

2) The deliverable obligation must be *pari passu* with the reference obligation. *Pari passu* is determined at the trade date (the date the swap contract was entered into). In particular, if after a restructuring an obligation that used to be a *pari passu* is subordinated, the obligation may still be deliverable.

3. RESTRUCTURING CASES

We discuss three cases where the restructuring clause was triggered and a fourth case where restructuring may soon occur. There are three particular issues on which we focus.

1) The shape of the curve of bond prices after the restructuring.

2) In all three restructuring cases, only bank loans were restructured.

3) The problem of distinguishing a restructuring and a refinancing: is a restructuring a modification of an existing loan or a roll-over into a new loan?

3.1 Recent restructurings

Identifying restructurings is an industry-wide problem. Moody's, for example, only classifies some restructurings as defaults to be included in their ratings and default data. In particular, Moody's only considers *involuntary distressed exchanges* as defaults[2]. Unfortunately, we have not been able to obtain reliable historical data that specifically identify restructurings. The main problem is that restructurings received little attention prior to the development of the default swap market, and in particular before the Conseco restructuring in autumn 2000. We manually examined a number of major default swap triggering events occurring after the Conseco case and only identified two additional restructurings: Xerox and Solutia.

3.2 Conseco

On 22 September 2000, Conseco announced an agreement with a 25-bank consortium to resolve issues surrounding $1.4 billion of bank debt coming due on that day. Over the preceding four years, Conseco's bank and public debt had increased to $5.9 billon and the agreement called for a reduction of $1.52 billon of bank debt and $1.56 billon of public debt over the following three years. To begin the debt reduction Conseco identified $2 billon worth of assets to be sold, of which $700 million had already been realized. The agreement included immediate repayment of $650 million to the banks, an extension of $571 million of bank debt until year-end 2001 to be financed by asset sales, and an acceleration of $300 million on $1.5 billon of bank debt due in September 2003 but with the option to extend the remaining $1.2 billon of bank debt until 2005. The rating agencies interpreted this financial restructuring plan positively and focused mainly on the fact that the plan improved liquidity and resolved concerns about the maturing bank debt[3].

The extension of the $571 million of bank debt until year-end 2001 clearly qualifies as a restructuring. The fact that $1.2 billion of the bank debt maturing in September 2003 was modified to include an option to extend until 2005 may also qualify as a restructuring. The latest maturity of restructured debt would then be 2005. However, protection sellers may argue that the option to extend was compensated by the $300 million acceleration, and therefore does not qualify as a restructuring. The legal issues are not clear.

Conseco, Inc. had ten bonds trading during September 2000, according to Lehman's index database. The reaction of the bond market to the restructuring announcement can be seen from the prices in Figure 2, where the two dates shown are before and after the restructuring announcement.

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[2] *For discussion see "Understanding The Risks In Credit Default Swaps", Moody's Special Report, 16 March 2001.*

[3] *See announcements made by A.M. Best and Fitch on 25 September 2000, and by S&P on 26 September 2000.*

Figure 2. Conseco Inc. bonds outstanding when the restructuring was announced on 22 September 2000

Maturity	15 Dec 2000	15 Jun 2001	21 Jun 2001	15 Oct 2002	10 Feb 2003	15 Feb 2003	9 Feb 2004	15 Dec 2004	15 Jun 2005	15 Oct 2006
Coupon	7.875	6.40	7.60	8.50	6.40	8.125	8.750	10.50	6.80	9.00
Outstanding (m)	150	550	118.9	450	250	95.3	800	24.55	250	500
Currency	USD	USD	USD	USD	USD	USD	USD	USD	USD	USD
Options / Sinking fund	None	Call/Put	None	Call	Call	None	Call	None	Call	Call
Price (31 Aug 2000)	94.20	90.86	91.50	66	62	65.19	62	55.98	62	61
Price (30 Sep 2000)	97.44	92.32	92.88	82	72	78.35	71	73.18	66	69
Excess return[4](%)	3.45	1.59	1.60	23.36	16.05	20.28	14.65	30.13	6.23	12.62

Assume that only the extension until 12/31/2001 qualifies as a restructuring and consider a default swap that matures in one year. Under Mod-R, the restructuring maturity limitation date would be 12/31/2001 and the cheapest-to-deliver (among the bonds listed) are the two June 2001 bonds priced around 92-93. Under Mod-Mod-R, the restructuring maturity limitation date would be 3/22/2003 for non-restructured obligations and 9/22/2005 for restructured obligations. None of the listed bonds were restructured, so the cheapest-to-deliver is the February 2003 bond priced at 72. When considering Old-R contracts, all listed bonds can be delivered and the cheapest-to-deliver is the June 2005 bond priced at 66.

3.3 Xerox

Market participants had long feared the October 2002 maturity of Xerox's $7 billion revolving credit facility[5]. On 21 June 2002, Xerox announced a renegotiation of the facility, paying down $2.8 billion and refinancing the remaining $4.2 billion. The financing consisted of three loans totaling $2.7 billion (maturing on 9/15/2002 to 4/30/2005) and a $1.5 billion revolver[6].

According to articles in *Derivatives Week* on 8 September and 22 December 2002, default swap sellers were disputing the occurrence of the credit event both for lack of publicly available information and by arguing that the refinancing did not *"directly or indirectly result from a deterioration in Xerox's creditworthiness or financial condition"*. On the other hand, default swap buyers argue that *"Xerox had no chance of repaying the loan and was forced into a restructuring"*.

The experience highlights the problem of determining whether a refinancing is a modification of an existing loan or roll-over into a new loan. The borrower may be indifferent and may easily be persuaded by the banks to classify the refinancing as a restructuring that can trigger default swaps held by the banks.

Using the Lehman index database we extracted prices for seven Xerox Corp. bonds trading during June 2002. Prices are shown in Figure 3.

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[4] *Excess return is excess return over what can be attributed to changes in Treasury rates. The calculation method used is described in "A New Method of Excess Returns Computation", Index Report, September 2000.*

[5] *See, for example, the 12 March 2001 issue of* Distressed Digest *published by our distressed research team.*

[6] *For details see, for example, equity research analyst Caroline Sabbagha's 24 June 2002 report on Xerox.*

Figure 3. Xerox Corp. bonds outstanding when the restructuring was announced on 21 June 2002.

Maturity	15 Nov 2003	15 Dec 2003	4 Feb 2004	1 Aug 2004	3 Dec 2004	15 Jan 2009	15 Jan 2009	1 Apr 2016	15 Nov 2026
Coupon	5.5	5.25	3.5	7.15	5.25	9.75	9.75	7.20	6.25
Outstanding (m)	600	250	300	200	750	600	250	250	350
Currency	USD	USD	EUR	USD	EUR	USD	EUR	USD	USD
Options / Sinking fund	None	None	None	None	None	Call	None	None	Put
Price (31 May 2002)	92.5	91.5	85	91	83	93	87	75	92
Price (28 Jun 2002)	85.5	85	76	83	74	82	77	73	89
Excess return (%)	-7.67	-7.10	-10.93	-8.90	-11.10	-12.10	-11.82	-3.67	-3.3

Outstanding amounts and prices are in the currency indicated. The USD/EUR exchange rate was 0.9321, 0.9700 and 0.9914 on 31 May 2002, 21 Jun 2002 and 28 Jun 2002 respectively.

In Figure 3, there are bonds denominated in euros as well as US dollars. ISDA documentation contains an option to specify the currency of deliverable obligations. It is common not to specify any particular currency, in which case the default is the currencies of the G7 countries and Switzerland. It is interesting to note the lower prices (but also lower coupons) on the two shorter term euro bonds, which raises the question of whether US dollar bonds have been bid up by default swap holders looking to purchase obligations to settle contracts that require US dollar deliverables. On the other hand, there is also the possibility that the euro bond prices are stale or that they may not be delivered for other reasons.

Assume that the latest maturity of a restructured obligation is 4/30/2005, that is about 34 months from the restructuring date. In this case the Mod-R maturity limitation approximation is exact and the maturity limitation under Mod-R is the same as the Mod-Mod-R maturity limit on non-restructured obligations. Ignoring the euro bonds, the cheapest-to-deliver under Mod-R is the August 2004 bond priced at 83. This is also the cheapest non-restructured bond to deliver under Mod-Mod-R. Under Old-R, the cheapest to-deliver is the April 2016 bond priced at 73.

3.4 Solutia

Solutia announced on 19 July 2002 that it had reached an agreement with its banks to extend the maturity of a 5-year maturing revolving loan facility for two years, and to reduce the facility from $800 million to $600 million. The $200 million paid to the banks came from a bond issued two weeks earlier. This was viewed as being detrimental to the wealth of existing obligation holders.

At the end of June 2002, there were five Solutia bonds in Lehman's index database and the new bond mentioned above. Prices are given in Figure 4. The 2005 bond is issued by Solutia Europe. The remaining four bonds are issued by Solutia Inc.

Figure 4. Solutia Inc. bonds outstanding when the restructuring was announced on 19 July 2002

Maturity	15 Oct 2002	14 Feb 2005	15 Jul 2009	15 Oct 2027	15 Oct 2037
Coupon	6.50	6.25	11.25	7.375	6.72
Outstanding (m)	150	200	223	300	150
Currency	USD	EUR	USD	USD	USD
Options / Sinking fund	None	None	None	Call	Call/Put
Price (28 Jun 2002)	100.0	86	NA	62.3	91.99
Price (31 Jul 2002)	99.59	78	88.5	60	80
Excess Return (%)	0.00	-9.54	NA	-5.38	-13.79

Outstanding amounts and prices are in the currency indicated. The USD/EUR exchange rate was 0.9914, 1.0136 and 0.9778 on 28 Jun 2002, 19 Jul 2002 and 31 Jul 2002 respectively.

The loan facility was extended until 7/19/2004. For default swaps maturing before that date, this is the restructuring maturity limitation date under Mod-R. Under the Mod-Mod-R, the date is 1/19/2005 for non-restructured obligations. However, this six-month difference is not enough to make the February 2005 deliverable. Of the bonds in the table, only the October 2002 bond may settle Mod-R and Mod-Mod-R contracts. Old-R contracts, on the other hand, can be settled with the October 2027 bond priced at 60.

3.5 Goodyear

On 5 March 2003, it was reported that Goodyear Tire and Rubber had obtained a $1.3 billion conditional asset-backed credit facility that would come into effect if/when Goodyear finishes negotiating changes in existing loan agreements. The reports were not specific about the required changes but the possibility of a restructuring event was apparent.

On 1 April 2003, it was reported that Goodyear had reached an agreement to restructure and refinance its loans. $2.9 billion of existing facilities were replaced by a $750 million secured revolving credit facility due in 2005, a $645 million secured US term facility due in 2005, a $650 million secured European facility due in 2005, and a $1.3 billion asset-backed facility due in 2006. According to a Reuters report *"the company said its restructured credit agreements replace facilities that generally have shorter maturities"*.

According to a default swap trader, the company paid down existing maturing facilities and replaced them with longer dates ones. Consequently a default swap restructuring event was deemed not to have occurred.

Prices of Goodyear bonds taken from our index database are shown in Figure 5.

Figure 5. Goodyear bonds outstanding when the possibility of a restructuring was announced on 5 March 2003

Maturity	15 Mar 2003	6 Jun 2005	17 Mar 2006	1 Dec 2006	15 Mar 2007	15 Mar 2008	15 Aug 2011	15 Mar 2028
Coupon	8.125	6.375	5.375	6.625	8.50	6.375	7.857	7.00
Outstanding (m)	300	400	200	250	300	100	650	150
Currency	USD	EUR	CHF	USD	USD	USD	USD	USD
Options / Sinking fund	Call	None	None	Call	Call	Call	Call	Call
Price (28 Feb 2003)	99.728	74	62.1	73	75	67.212	67	60
Price (31 Mar 2003)	100	78	73	75	77	69.459	71	65
Excess Return (%)	0.47	5.71	16.70	3.34	3.38	4.04	7.18	9.52

Outstanding amounts and prices are in the currency indicated. The USD/EUR exchange rate was 1.0820 and 1.0973 on 28 Feb 2003 and 5 Mar 2003 respectively. The CHF/USD exchange rate was 1.3521 and 1.3294 on those dates.

3.6 Lessons

Even from this small sample we see that there can be considerable value in the delivery option. Figure 6 shows the recovery under the different clauses.

Figure 6. Recovery on default swaps with three different restructuring clauses

	Mod-R	Mod-Mod-R	Old-R
Conseco	93%	72%	66%
Xerox	83%	83%	73%
Solutia	99.4%	99.4%	60%

Recovery is the cost (as a percentage of par) of the cheapest-to-deliver obligation under the particular contract. Only bonds included in a Lehman Brothers index were considered.

Unfortunately, the sample is too small to discern any general pattern. This is not surprising, as restructuring can occur for many reasons. A company, while basically healthy, may have liquidity issues forcing it to restructure or a company may be in serious financial trouble when it restructures. A restructuring may reduce the default risk for short-term claim holders, although the terms of the restructuring may alter possible recovery if default does occur.

4. PRICING

In this section we present a simple framework to analyze the pricing implications of the four different restructuring clauses. Before describing the pricing model, we consider the relative rankings of the different spreads.

4.1 Basics

Let S_{NR}, S_{MR}, S_{MMR}, and S_{OR} be the default swap spreads under the four different restructuring clauses, No-R, Mod-R, Mod-Mod-R, and Old-R. A simple argument implies that

$$S_{NR} \le S_{MR} \le S_{MMR} \le S_{OR}$$

The spread for No-R, S_{NR}, will be the lowest of the four spreads, since in all the other three clauses, the protection buyer has the option but not the obligation to settle the contract if a restructuring occurs. The spread for Old-R, S_{OR} will be the largest of the four spreads, because any obligation that can be delivered under Mod-R and Mod-Mod-R can also be delivered under Old-R. Finally, we saw in the previous section that any obligation that may be delivered under Mod-R may also be delivered under Mod-Mod-R, which implies that the spread for Mod-Mod-R should be greater than or equal to the spread for Mod-R.

4.2 A simple model

In the absence of restructuring as a credit event, we could price a default swap using the term structure of "risk-neutral" default probabilities. Default would then be the usual definition and include failure to pay and bankruptcy but not restructuring. The article "Valuation of Credit Default Swaps" included elsewhere in this publication describes how this can be done.

When a company files for Chapter 11 or fails to make debt payments it is usually in dire financial trouble and the company's debt will trade on the expected recovery value. In particular, two debt obligations with the same seniority will trade at roughly the same price even if one obligation matures in two years and the other obligation matures 20 years later. All obligations that may be delivered to settle a default swap contract will trade at roughly the same price, and there is little reason to specifically model which obligation is the cheapest-to-deliver. This is not the case for restructurings. As we have seen in section 3, although a company that restructures its debt is unlikely to be entirely financially healthy, it may well avoid bankruptcy (at least for a few years). For this reason, maturity matters when pricing the company's debt and the deliverable obligations will usually not trade at the same price. It is therefore important to explicitly model which obligation is the cheapest-to-deliver.

We modify the well-known Jarrow-Turnbull credit pricing framework and directly incorporate the possibility of restructuring as well as default. We model the occurrence of a restructuring the same way as the occurrence of a default, by specifying a hazard rate for a jump process.

The model takes as input:

1) A curve of default-free interest rates: $\{r_t\}$.
2) A curve of risk-neutral one-year conditional probabilities of default: $\{\lambda_t\}$.
3) A curve of risk-neutral one-year conditional probabilities of restructuring: $\{q_t\}$.
4) A default recovery rate for the *pari passu* obligations: R.
5) A curve of risk-neutral one-year conditional probabilities of default after a restructuring: $\{\theta_t\}$.
6) A minimum coupon on debt outstanding after a restructuring: c_{min}.
7) A maturity limit on debt outstanding after a restructuring: T_{max}.

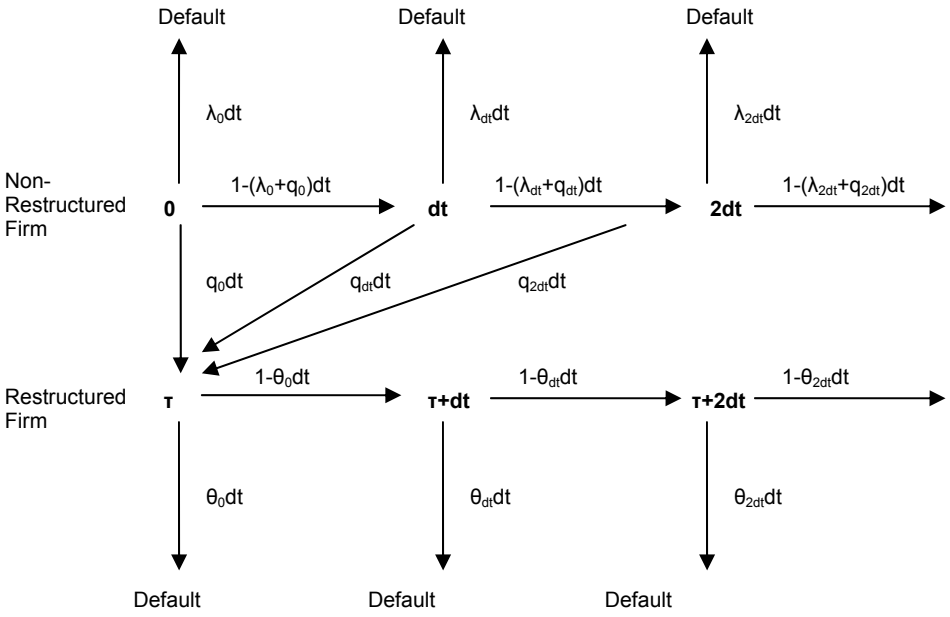
The model is based on the following simplifying assumptions:

1) A restructuring can only occur once in the life of a default swap.

2) After a restructuring, the cheapest-to-deliver is a bond maturing with a coupon of c_{min} and a maturity of $\max\{T_{max}, T\}$, where T is the remaining time to maturity of the default swap.

3) Under Mod-R we use T_{max} = 30 months, under Mod-Mod-R we use T_{max} = 60 months, and under Old-R we use T_{max} = 360 months.

Figure 7. Pricing model with both default and restructuring included as credit events



The firm starts as a non-restructured firm. Next period the firm will default, restructure or survive as a non-restructured firm. τ is the time of restructuring. Each arrow represents the passing of a time period of length dt. The model assumes that a firm can only restructure once and that the default probabilities after a restructuring are the same no matter when the firm restructured.

Figure 7 illustrates how the model works. It is straightforward to solve the model numerically using standard backwards-induction option pricing techniques[7]. To find the spread on a default swap with a particular restructuring clause take the following steps:

1) For a swap with a 5-year maturity, divide the 5-year period into a number of discrete time points, say of length dt. Then calculate the probabilities of restructuring and default over each time period as illustrated in Figure 7.

2) For every time point, find the price of the cheapest-to-deliver bond. This is done using

   i) the curve of default-free interest rates starting from the (time) point of the restructuring,

   ii) the curve of probabilities of default after a restructuring,

[7] *Because all hazard rates are constant, solution of the model amounts to nothing more than calculation of a few sums. With stochastic hazard rates the model becomes more complicated and we would need to build a lattice.*

iii) the default recovery rate, and

iv) the scheduled payments for the set of deliverable obligations.

3) Price the swap by backwards induction using the probabilities from step 1, the default recovery rate and the cheapest-to-deliver bond from step 2. It must be taken into account that the protection buyer has the option not to settle the contract at restructuring and instead wait for a possible future default that may give a higher protection payment.

4.3 Calibration

Determining the input parameters is the final step before reporting results. This is also the most difficult step. We need to determine:

1) the probabilities of default before a restructuring, $\{\lambda_t\}$,

2) the probabilities of restructuring, $\{q_t\}$, and

3) the probabilities of default after a restructuring, $\{\theta_t\}$.

We suggest thinking about the probabilities of restructuring as a multiple of the probabilities of default. That is, the conditional probabilities of restructuring are $q_t = c * \lambda_t$, for all t, where c is a constant. In Figure 8, below, we consider three different values for c, c = 2, 1 and 0.5. We use this specification because factors that increase the probability of default will usually also increase the probability of restructuring.

When picking the multiple c, the analyst should focus on factors that mainly affect the probability of restructuring. For example, whether the firm has large amounts of bank debt and how complex its capital structure is (both are important determinants of how easy it will be to reach a restructuring agreement).

For choosing the probabilities of default after a restructuring, $\{\theta_t\}$, we refer to the curve of bond prices observed in the three restructuring cases. For reasons discussed in section 3, we suggest using an upward sloping curve of probabilities.

4.4 Results

In Figure 8 we report the default swap spreads for a number of different parameter cases. In all cases, we have fixed the curve of default-free interest rates to be upward sloping with a short rate of 3% and a long rate of 6%[8]. We use a default recovery rate of R = 50% and a minimum coupon of c_{min} = 6%. The results are also based on constant probabilities of default ($\lambda_t = \lambda_0$, for all t) and a curve of probabilities of default after restructuring where the long-run probability is the double of the short-run probability[9].

Suppose that for No-R the spread is 100bp and the restructuring/default probability ratio is 2. We then compute the spreads for the Mod-R, Mod-Mod-R and Old-R. Next, we alter the ratio to 1. We keep the spread for No-R fixed at 100bp and we alter the default probability, λ_0, so that No-R remains at 100bp, and then we compute the new level of spreads for the Mod-R, Mod-Mod-R and Old-R contracts.

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[8] *We use $r_t = 3\% \cdot (2-\exp(-0.1 \cdot t))$, for all $t \geq 0$.*

[9] *We use the $\theta_t = \theta_0 \cdot (2 - \exp(-0.1 \cdot t))$, for all $t > 0$.*

Figure 8. Default swap spreads (in bp) under different restructuring clauses

1st year default probability after restructuring[10]	5%				10%				15%			
Restructuring/default probability ratio	**2**	**1**	**0.5**		**2**	**1**	**0.5**		**2**	**1**	**0.5**	
Restructuring Clause	Spread	Spread	Spread	Recovery	Spread	Spread	Spread	Recovery	Spread	Spread	Spread	Recovery
No-R	100	100	100	NA	100	100	100	NA	100	100	100	NA
Mod-R	100	100	100	99%	104	102	101	93%	107	104	102	88%
Mod-Mod-R	103	102	101	97%	115	109	105	86%	122	114	108	78%
Old-R	150	127	114	82%	159	134	119	71%	155	134	119	65%
No-R	200	200	200	NA	200	200	200	NA	200	200	200	NA
Mod-R	201	201	200	99%	209	206	203	93%	215	209	205	88%
Mod-Mod-R	211	206	203	97%	239	223	212	86%	252	232	218	78%
Old-R	321	264	233	82%	334	277	241	71%	321	273	241	65%
No-R	300	300	300	NA	300	300	300	NA	300	300	300	NA
Mod-R	321	310	305	99%	330	317	309	93%	336	322	312	88%
Mod-Mod-R	344	322	311	97%	379	345	324	86%	393	356	331	78%
Old-R	521	412	356	82%	528	428	368	71%	501	420	367	65%

Recovery is the cost (as a percentage of par) of the cheapest-to-deliver obligation under the particular contract.

The spread between Old-R and Mod-Mod-R is larger than the spread between Mod-R and Mod-Mod-R[11]. The spread between No-R and Mod-R is relatively small, but the spread between Mod-Mod-R and Old-R can be substantial. The market rule of thumb of adjusting the spread between the different contracts by a fixed percentage seems to be inappropriate. Given the difficulty in calibrating the model, we have arbitrarily chosen the parameter values and consequently the spread differences presented in figure 8 may not be realistic.

4.5 Back-of-the-envelope calculations

To gain some intuition into the numbers in Figure 8, consider the case where the spread on the No-R contract is 300bp, the restructuring/default probability ratio is 2, and first year default probability after restructuring is 15%. Let us go through a very rough back-of-the-envelope type of calculation to explain the order of magnitude of the Old-R spread, which is 501bp in Figure 8, compared to the No-R spread.

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[10] *To be precise the numbers in this row are* θ_0 and the instantaneous (risk-neutral) hazard rates are $\theta_t = \theta_0 \cdot (2 - \exp(-0.1 \cdot t))$, *for* $t>0$. *That is, we are using an upward slopping curve of hazard rates where the long-run hazard rate is the double the short-run hazard rate. The risk-neutral probability of default within the first year is 1-exp(-θ'), where θ' is the integral from 0 to t of* θ_t.

[11] *It is important to remember that the Old-R spreads in Figure 8 are based on the assumption that the firm will have 30-year debt outstanding at the time of restructuring. It is important to evaluate in each particular case whether this is a reasonable assumption.*

First consider the No-R contract. For this contract restructuring is not a defined credit event but does affect the probabilities of default. There are two default events to consider.

1) No restructuring and then a default with a loss of 0.5. This occurs with a probability of (1–2p)·p, where p is the probability of default and (1–2p) is the probability of no restructuring. The expected loss is (1–2p)·p·0.5.

2) Restructuring and then a default. After a restructuring the new probability of default is approximately 18% (remember we use an upward sloping curve of hazard rates, see footnote 10 for details; the average hazard rate over the first five years following a restructuring is approximately 18%) and the expected loss is 2p·18%·0.5 for this event, where the term 2p is the probability of restructuring.

The total expected loss should equal the spread of 300bp, ie,

$$(1-2p)\cdot p\cdot 0.5 + 2p\cdot 18\%\cdot 0.5 = 3\%$$

The equation gives an implied probability of default p = 4.74%[12].

Now consider the Old-R contract. We either have default and no restructuring with probability p·(1–2p) and a loss of 0.5 or no default and restructuring with probability (1- p)·2p and a loss of 0.35. Note that we ignore the joint event of restructuring and default occurring together. The spread is

$$p\cdot(1-2p)\cdot 0.5 + (1-p)\cdot 2p\cdot 0.35 = 5.27\%$$

This compares with a spread for Old–R in the Figure 8 of 501bp.

Other types of back-of-the-envelope calculations can be made. Consider the same case as above. Assume that only two events can occur: 1) The firm goes straight to default (without restructuring first) with a hazard rate of 4.2%, or 2) it restructures with a hazard rate of 8.4%. If we know what the protection buyer's position is worth in each event under the two contracts we can approximate the spreads on the contracts. For the Old-R contract, the payment at default is 0.5 and the payment at restructuring is 0.35, and the spread should be approximately 4.2%·0.5 + 8.4%·0.35 = 5.04%, which is close to the 501bp in Figure 8. For the No-R contract the payment at default is also 0.5 but what is the value of the position to the protection buyer right after a restructuring has occurred? If this value is 0.105, the spread on No-R contracts would be 4.2%·0.5 + 8.4%·0.105 = 2.98%, which is close to the 300bp in Figure 8. Can we justify a value of 0.105? Here is an attempt: Given that a restructuring has occurred during the five years of the contract, it will on average have occurred with 2.5 years remaining (because of the flat hazard rate of restructuring). After a restructuring, the hazard rate of default over the remaining 2.5 years is about 17% and the loss given default is still 0.5, so the spread after a restructuring on a new 2.5 year swap (with a market value of zero) is approximately 17%·0.5 = 850bp or 550bp more than the protection buyer is currently paying. In other words, entering into an offsetting position will provide 550bp for about 2.5 years or until default, whichever comes first. To value this stream of payments we must know the PV01 of the stream, which in this case is about 1.9[13]. The No-R protection buyer would have

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[12] *Note this is a quadratic with two roots. We have taken the root that is close to the range of acceptable default probabilities.*

[13] *It is a bit more cumbersome to justify that the PV01 is 1.9. The PV01 can be approximated by $(1+x)^{-1} + (1+x)^{-2} + (1+x)^{-3} + (1+x)^{-4} + (1+x)^{-5}$, where x = 0.5·(17% + 3%) and 3% is the default free rate. This calculation produces 1.90 but is not exact. See the article "Valuation of Default Swaps" elsewhere in this publication for details on how to calculate PV01 and in general for details on how a default swap should be marked-to-market.*

a profit at restructuring of approximately $0.055 \cdot 1.9 = 0.1045$, which is close to the profit of 0.105 we wanted to justify.

There are many reasons why the back-of-the-envelope numbers differ from the numbers in Figure 8. We ignore the maturity of the contract, the term structure of interest rates and the random timing of the events.

5. CONCLUSION

In this paper we describe the differences between Old-R, Mod-R and Mod-Mod-R and present a simple model that can used to price these different types of contracts. The major difficulty is the calibration of the model. To apply the model to individual firms we need to specifically model the maturity of the bonds the firm is expected to have outstanding after a restructuring, and we need to have a view on both the likelihood of a restructuring vs a default and the probabilities of default after a restructuring.

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