Correlation Vehicles

Techniques for trading equity correlation

Correlation reflects the difference between index and single stock volatilities

Short correlation trades perform well, exploiting the relative richness of index volatility

Overview

In this note we compare alternative vehicles for trading equity correlation, how well they have performed, and what drives their p/l.

In general, short correlation trades – often in the form of dispersion trades – have performed well, with return/risk ratios above 2. Such trades are usually constructed by selling index variance swaps and buying single stock variance swaps. They seek to take advantage of the relative richness of index volatility without being outright short volatility.

However, depending on trade construction, a short correlation trade can actually be an efficient way to own volatility.

Figure 1: Euro Stoxx 50 3-month implied and realised correlation ... implied tends to trade above realised
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Introduction

Equity correlation measures how much stock prices tend to move together. It provides the link between the volatility of an index and the volatilities of component stocks; described approximately by the formula:

\[
\text{Index Volatility} \approx \sqrt{\text{correlation}} \times \text{Average Single Stock Volatility}
\]

This formula shows that correlation is an important factor in index volatility: for example, backtesting shows that changes in index volatility are at least 50% driven by changes in correlation.

By examining the relative levels of implied index and implied single-stock volatilities, it is possible to back out the level of forward-looking correlation being priced in by the market. As with implied index volatility, this implied correlation tends to trade at a premium to that actually delivered (Figure 2).

Like volatility, correlation can itself be treated as an asset class and traded in its own right. Selling correlation has historically been profitable, capitalising on the relative richness of index volatility. The spread between index implied and realised volatility has been generally positive and has consistently exceeded the spread between average implied and realised single-stock volatility. To take advantage of both of these opportunities we can sell index volatility to profit from its richness, whilst buying single-stock volatility in order to hedge out some or all of the short volatility exposure.

Why trade correlation?

- A short correlation position can be a good source of alpha, since implied correlation usually trades at a premium to that delivered
- Trading correlation allows diversification of a portfolio – returns from correlation are somewhat anti-correlated with index returns
- Correlation allows a view to be taken on the relative moves of index volatility in comparison to single-stock volatility
- Long correlation (implied or realised) can be a good hedge against market risk

Table 1: Returns and volatility of strategies (in vegas) and risk-return ratios.

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<th>3m</th>
<th>6m</th>
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<tbody>
<tr>
<td><strong>Long vanilla dispersion</strong></td>
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<tr>
<td>Return (annualised)</td>
<td>8.6%</td>
<td>4.8%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Volatility (of return)</td>
<td>4.5%</td>
<td>2.0%</td>
<td>1.3%</td>
</tr>
<tr>
<td>Risk-Return</td>
<td>1.9</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td><strong>Long correlation-weighted dispersion</strong></td>
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<tr>
<td>Return (annualised)</td>
<td>8.8%</td>
<td>2.9%</td>
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<tr>
<td>Volatility (of return)</td>
<td>3.4%</td>
<td>1.3%</td>
<td>0.5%</td>
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<tr>
<td>Risk-Return</td>
<td>2.6</td>
<td>2.2</td>
<td>2.6</td>
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<tr>
<td><strong>Short index variance (for comparison)</strong></td>
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<tr>
<td>Return (annualised)</td>
<td>3.8%</td>
<td>-0.1%</td>
<td>-2.2%</td>
</tr>
<tr>
<td>Volatility (of return)</td>
<td>9.9%</td>
<td>5.4%</td>
<td>2.7%</td>
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<tr>
<td>Risk-Return</td>
<td>1.5</td>
<td>0.0</td>
<td>-0.8</td>
</tr>
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Source: JPMorgan. Results assume 3% bid-offer on dispersion trades and 1.5% bid-offer on variance swaps. Data August 2000 to date

Returns from short correlation trades have been largely positive over the last few years (Table 1). For example, selling 6-month correlation via correlation-weighted dispersion has generated an average annual return of 2.9 vegas, with a risk-return ratio of 2.2. However, over the last two years, absolute levels of returns from dispersion trades have been markedly reduced, suffering from the reduction in levels of volatility. Despite this, risk-return ratios of correlation-weighted trades have managed to remain roughly stable due to a corresponding reduction in the volatility of the strategy. See pages 27-29 for details.
Vehicles for trading correlation - summary

This section summarises the principal vehicles for trading correlation and outlines their characteristics. This note continues by discussing the fundamentals behind correlation (pages 5–9). We investigate correlation trading vehicles in detail and dissect their p/l on pages 10–26. Returns and risks of trading correlation are reviewed on pages 27–29.

There are two principal vehicles for trading correlation, namely correlation swaps and variance dispersion trades. Correlation swaps give direct exposure to correlation, paying out on the difference between the strike of the swap, and the subsequent realised average pairwise correlation of a pre-agreed basket of stocks. Although correlation swaps are the easiest and most direct method of trading correlation, they have been less liquid than dispersion trades and harder to mark to market. Furthermore, due to the direct nature of their correlation exposure and the fact that implied correlation tends to trade at a premium to that delivered, the level of correlation which can be sold through a correlation swap has generally been lower than the level of implied correlation backed out from index and single-stock volatilities.

The most common vehicle for trading correlation has historically been the dispersion trade. This trade consists of taking opposing positions in the volatility of an index and the volatility of its constituents. Typically a dispersion trade would be effected through variance swaps known as a variance dispersion trade, where a long position would be constructed by selling variance on an index, and buying variance on its constituents. A long dispersion position will be short correlation but long the dispersion of volatilities of the index constituents.

The relative weightings of the constituent variance swaps are of crucial importance in a dispersion trade. In this note we discuss two different weighting schemas; a vanilla dispersion trade (the simplest) which has both correlation and outright volatility exposures, and a correlation-weighted dispersion trade, whose p/l more closely reflects changes in correlation.

It turns out that the vanilla dispersion trade has in fact been an efficient way to own volatility, in effect using the alpha available from short correlation to subsidise the cost of being long volatility. The short correlation exposure to some extent hedges the long volatility position but also allows the trade to profit from a decrease in correlation relative to volatility.

On the other hand, the correlation-weighted dispersion trade has profited more directly from moves in correlation, although there are other important drivers of p/l. In particular, though the trade is initially vega neutral, the p/l arising from correlation will be magnified or attenuated by prevailing levels of volatility, causing the position to develop a vega exposure as correlation moves.

Figure 3: vanilla dispersion payoff is closely related to long variance returns …

Figure 4: … whereas the correlation-weighted dispersion payoff is more related to correlation
What is Correlation?
The realised correlation of a pair of stocks measures how much the stock prices tend to move together. The realised correlation of an index is simply an average across all possible pairs of constituent stocks. There are two ways of computing this average, the results of which drive the payoffs of the two principal vehicles for trading correlation.

- **Average pairwise correlation**: This is a simple equally weighted average of the correlations between all pairs of distinct stocks in an index. It is the payoff of a **correlation swap**.

- **Index correlation**: This is a measure of correlation derived from the volatility of an index and its constituent single stocks. Realised index correlation turns out to be equal to the weighted sum of constituent pairwise stock correlations, where the correlations are weighted by both the stock weights and their respective volatilities (see Box 1, page 8 for details). This measure of correlation is the principal driver for the p/l of a **variance dispersion trade**.

In practice these two correlation measures are very similar. The difference between the two represents the spread of volatilities and correlations across the index. If either all volatilities are the same, or if all pairwise correlations are the same, the difference will be zero.

**Difference between correlation measures**
Whatever the maturity, index realised correlation will be greater than average pairwise correlation if the more volatile (and higher weighted) stocks in the index are more highly correlated. Through backtesting, we have found that for short-dated correlation, the two measures are almost identical, with the realised index correlation slightly greater than average pairwise correlation (Figure 5). This is supported by the fact that the average 3-month pairwise correlation has tended to be somewhat higher for the higher volatility stocks (Figure 6).

For 6-month and especially 1-year correlation, averaged pairwise correlation has been slightly greater than index realised correlation. However, the index realised correlation of a hypothetical equally weighted index is always very close to average pairwise correlation, suggesting, at least for longer-dated correlation, that it is the stock weightings rather than the volatility weightings that make the difference.
Implied Correlation

The advantage of using the index realised correlation measure is that it is possible to use the same method to back out an implied correlation measure from the implied index and single stock volatilities. This implied correlation measure represents the market’s expectations of future realised index correlation inherent in the implied volatility. Note that we can only compute this implied correlation if the index upon which it is based has actively traded volatility/options.

By using the correlation proxy examined in the following section, we can approximate implied correlation by squaring the ratio of implied index to average single-stock volatility. This approximation makes it easy to estimate the implied correlation of an index from implied volatilities available in the market.

Implied correlation tends to trade rich compared to realised correlation. This is a manifestation of the relative richness of index to single-stock volatility: the spread of index implied over realised volatility has generally exceeded the spread of single-stock implied over realised volatility (Figure 16). This richness is in part driven by structural forces of supply and demand within the equity derivatives market. For example, investors purchasing index puts to protect their downside creates demand for index volatility, whereas the selling of covered calls to earn alpha increases the supply of single-stock volatility.

There is another kind of implied correlation which is the strike of a correlation swap. Since the payoff of such a swap is equal to the average pairwise correlation and this is usually very close to the realised index correlation, the strike of a correlation swap should in theory trade close to the implied correlation described above.

However, unlike index implied correlation, correlation swap implied correlation cannot be inferred from other market data and is available only in the form of a quoted strike for a correlation swap. Such quotes can differ substantially from index implied correlation levels as a result of market appetite and difficulties arising from hedging and replication. From now on, unless stated otherwise, by implied correlation we will mean the correlation backed out from implied index and single-stock volatility and not the strike of a correlation swap.
The Correlation Proxy

Correlation is correctly calculated by the formula
\[ \rho_{ii} = \frac{\sigma_i^2 - \sum_i \omega_i^2 \sigma_i^2}{\left(\sum_i \omega_i \sigma_i\right)^2} \] (see Box 1 for details).

However, if the correlation is not too close to zero (in practice > 0.20) and the number of members of the index is large enough (more than about 20, but really what is important is that all the weights are relatively small), then the formula \( \rho_{ii} = \frac{\sigma_i^2}{\left(\sum_i \omega_i \sigma_i\right)^2} \) will be a good proxy for the correlation (Figure 10). (See “Fundamental relationship between an index’s volatility and the correlation and average volatility of its components”, Sebastien Bossu and Yi Gu, 2005.)

That is, the correlation is approximately equal to square of the ratio of index volatility to average single-stock volatility, allowing index volatility to be expressed in terms of correlation and average stock volatilities as follows:

\[ \text{Index Volatility} \approx \sqrt{\text{correlation} \times \text{Average Single Stock Volatility}}. \]

Example: the correlation proxy

In the Euro Stoxx 50, on 12 May 2005, average 3-month realised single-stock volatility was 16.3%, with 3-month realised correlation at 0.43. Multiplying 16.3% by the square-root of correlation gives an estimate for the realised index volatility of 10.7%, close to the true value of 10.8%.

In practice the approximation given by the correlation proxy is widely used. It is accurate except in times of very low correlation or for small indices. For example, if stock correlations were zero, current index volatility would be 2.5% not the 0% indicated by the proxy. Note, the correlation proxy always overestimates correlation.

This correlation proxy can be calculated in exactly the same way for implied correlation as for historic correlation, demonstrating that implied correlation is approximately equal to the square of the ratio of implied index volatility to average implied single-stock volatility.

The formulas for correlation and the correlation proxy can be used in reverse to predict a value for index volatility given values for correlation and average single-stock volatility (as in the above example). The relationship between correlation and volatility described by either the formula or the proxy is non-linear: when correlation is low, an increase in correlation will cause a greater increase in volatility than when correlation is high (Figure 9).

Figure 9: Volatility as a function of correlation: Euro Stoxx 50 weights and volatilities as of 01/04/2005

Figure 10: 6-month Euro Stoxx 50 realised correlation and proxy

Source: JPMorgan
Box 1: Calculating correlation

Index variance is described by the equation:

\[ \sigma_i^2 = \sum_i \omega_i^2 \sigma_i^2 + 2 \sum_{i<j} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}, \]

where \( \omega_i \) and \( \sigma_i \) are respectively the weight and volatility of the \( i^{th} \) stock in the index and \( \rho_{ij} \) is the pairwise correlation of the \( i^{th} \) and \( j^{th} \) stocks.

Assuming an index of \( N \) stocks, average pairwise correlation is given by

\[ \rho_A = \frac{2}{N(N-1)} \sum_{i<j} \rho_{ij}. \]

Index realised correlation is computed by solving the index variance equation for a single value of \( \rho \): giving

\[ \rho_H = \frac{\sigma_i^2 - \sum_i \omega_i^2 \sigma_i^2}{2 \sum_{i<j} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}}. \]

Using the identity \( \left( \sum_i \omega_i \sigma_i \right)^2 = \sum_i \sum_j \omega_i \omega_j \sigma_i \sigma_j = \sum_i \omega_i^2 \sigma_i^2 + 2 \sum_{i<j} \omega_i \omega_j \sigma_i \sigma_j \) we can write:

\[ \rho_H = \frac{\sigma_i^2 - \sum_i \omega_i^2 \sigma_i^2}{\left( \sum_i \omega_i \sigma_i \right)^2 - \sum_i \omega_i^2 \sigma_i^2}, \]

or alternatively \( \rho_H = \frac{\sum_{i<j} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}}{\sum_{i<j} \omega_i \omega_j \sigma_i \sigma_j}. \)

The implied correlation \( \rho_I \) is calculated in exactly the same way as \( \rho_H \), replacing realised volatilities \( \sigma_i \) and \( \sigma_I \) by the corresponding implied volatilities throughout the equation.

Note the interpretations of \( \left( \sum_i \omega_i \sigma_i \right)^2 \) and \( \sum_i \omega_i^2 \sigma_i^2 \). The first is the square of the average (weighted) single-stock volatility. The second is what the variance of the index would be with zero correlation. So we have two interpretations of the index realised correlation, described by the two forms above. The first shows that realised correlation is the ratio of index variance minus uncorrelated variance to average single stock volatility squared minus uncorrelated variance. The second form shows that realised correlation is the weighted sum of pairwise correlation, where the correlations are weighted by both the stock weights and their volatilities.

The difference between \( \rho_H \) and \( \rho_A \) can be given explicitly by considering the second form of \( \rho_H \).

\[ \rho_H - \rho_A = \frac{\sum_{ij} \left( \omega_i \omega_j \sigma_i \sigma_j - \overline{\omega_i \omega_j \sigma_i \sigma_j} \right) \rho_{ij}}{\sum_{ij} \omega_i \omega_j \sigma_i \sigma_j} \]

where \( \overline{\omega_i \omega_j \sigma_i \sigma_j} \) is the average of the product of all distinct pairs of weights and volatilities.
Correlation versus Volatility

Correlation is closely related to volatility (Figure 11 and Figure 12). It is not surprising that it is related to index volatility as correlation is a component of index volatility with the relationship approximately described by:

\[ \text{Index Volatility} \approx \sqrt{\text{correlation}} \times \text{Average Single Stock Volatility} \]

However, it is also true that correlation and average single-stock volatility are correlated (Figure 13). This is not entirely surprising either since correlation and volatility, both implied and historic, are driven by similar factors. Correlation measures how much stocks tend to move together and this is more likely in times of high volatility – for example during a sell-off prompted by some kind of negative surprise.

Note that whilst correlation has fallen since 2002/2003 and especially in recent months, both implied and realised correlation are still looking high relative to their history when compared to volatility. This could be because correlation is biased to fall further or perhaps because the underlying environment has changed and we should expect correlation to be higher, even in a low-volatility world (see page 30).

Figure 11: Euro Stoxx 50 6-month implied correlation and volatility

Figure 12: Euro Stoxx 50 6-month realised correlation and volatility

Figure 13: Correlation of 6m Euro Stoxx 50 realised correlation and 6m realised average single stock volatility has been strong since the beginning of 2002

Figure 14: On September 11 2001, implied correlation reached 1.00 as index implied volatility overtook average single-stock volatility

Implied correlation is especially sensitive to changes in volatility. This is probably due to the greater liquidity in trading index volatility compared to single-stock volatility. In September 2001, excess demand for put-protection at the index level combined with a lagged remarking of single stock volatility drove up implied correlation above its theoretical maximum of 1.00. Implied correlation then fell as single-stock volatilities caught back up with index volatility (see Figure 14).
Trading Correlation

Selling correlation has historically been profitable, capitalising on the relative richness of index volatility. The spread between index implied and realised volatility has been generally positive (Figure 15) and has consistently exceeded the spread between average implied and realised single-stock volatility. To take advantage of both of these opportunities we can sell index volatility to profit from its richness, whilst buying single-stock volatility in order to hedge out some or all of the short volatility exposure.

Figure 15: Implied index volatility has generally traded above historic volatility – except during periods of high volatility and …

There are two principal vehicles for trading correlation, namely correlation swaps and variance dispersion trades. Correlation swaps are the easiest and most direct method, but they are less liquid and harder to mark to market. The market for variance dispersion trades has been much larger than that for correlation swaps, although the correlation swap market is growing as the result of exotics desks trading correlation in this more direct form to hedge their correlation exposures.

Due to hedging difficulties and the direct nature of the correlation exposure, correlation swap strikes have tended to trade around the levels of correlation realised by the relevant basket of underlying stocks. For this reason, and because implied correlation generally exceeds that delivered, the level of correlation that can be sold via a correlation swap has tended to be lower than that sold via a variance dispersion trade.

Below we discuss the mechanics of correlation swaps and variance dispersion. We also dissect the p/l of a variance dispersion trade to identify the factors other than correlation which drive p/l.

Trading Correlation - Correlation Swaps

Correlation can be traded directly via a correlation swap. A correlation swap gives exposure to the average pairwise correlation of a pre-determined basket of stocks. The swap strike is the level of (average pairwise) correlation that is bought or sold, and is typically scaled by a factor of 100 (i.e. a correlation of 0.55 is quoted as a strike of 55).

The payout of a correlation swap is the notional amount multiplied by the difference between the swap strike and the subsequent realised average pairwise correlation on the basket of underlyings.

payout = notional × (realised average pairwise correlation – strike)
Correlation swap cashflows

Example: correlation swaps
Suppose that on October 1st 2004 we sell a 6-month correlation swap on an equally weighted basket of stocks consisting of the members of the Euro Stoxx 50 index. The strike of the correlation swap is 55, and the notional amount is €10,000

After six months, on April 1st 2005, we calculate the realised 6-month average pairwise correlation of the stocks in our basket as 0.42. Then the p/l is calculated as:

\[
P/L = \text{notional} \times (\text{strike} - \text{realised pairwise correlation})
\]

\[
= €10,000 \times (55 - 42)
\]

\[
= €130,000
\]

The advantages of a correlation swap over other forms of correlation trade are:

- The correlation swap gives direct exposure to the level of delivered (average pairwise) correlation with no dynamic hedging/replication required.

- A correlation swap can be set up on any basket of underlyings, not necessarily a traded index. The buyer and seller simply agree on the basket, the notional and the level of correlation traded (the strike) at the inception of the trade.

- Bid-offer spreads are much lower on correlation swaps than on dispersion trades. A spread of around 3 correlation points would be typical for a correlation swap, equating to between about 0.5 and 1 vega. In contrast the bid-offer spread on a dispersion trade is about 2 to 3 vegas.

Due to the direct nature of the correlation exposure the correlation swap strike has tended to trade at around the levels of correlation realised by the relevant basket of underlyings, as opposed to trading around the implied levels of correlation backed out from the index. As a result of this correlation swap strikes have tended to trade below index implied correlation. For example in late April 2005, 1-year index implied correlation was around 60 whereas correlation swap strikes were trading nearer to 45.
Variance Swaps

Variance swaps are the building blocks of variance dispersion trades, the most common form of dispersion trade. This section provides a summary of the most important properties of variance swaps, before going on to consider the various forms of variance dispersion trade in the following sections.

A variance swap is an OTC instrument offering investors direct exposure to the volatility of an underlying asset. The strike of the swap is the level of volatility bought or sold and is agreed at trade inception. The notional is typically expressed as a vega amount which is the average p/l for a 1% move in volatility. The true notional is the variance amount which is the vega amount divided by twice the strike. The p/l of the swap at expiry is equal to the variance amount multiplied by the difference between the strike squared and the realised variance.

The p/l for a (short) variance swap with strike $K$ and subsequent realised variance $\sigma^2$ is given by

$$p/l = N_{\text{Vega}} \times \left( \frac{K^2 - \sigma^2}{2K} \right) = N_{\text{Variance}} \times \left( K^2 - \sigma^2 \right)$$

where $N_{\text{Vega}}$ is the vega notional and $N_{\text{Variance}}$ is the variance notional.

By convention, volatility is scaled by a factor of 100 (i.e. a strike of 20 represents a volatility of 20%)

Payoffs from variance swaps are convex in volatility. That is, a long position will gain more from an increase in realised volatility than it will lose for a corresponding decrease. For this reason variance swap strikes trade at a premium to a pure volatility swap. In addition a variance swap can be statically hedged by a (continuous) portfolio of options across a range of strikes, but more strongly weighted towards the downside strikes. Thus (due to positive skew) the strike of a variance swap tends to trade above at-the-money implied volatility – typically around the 90-95% strike level depending on maturity. (See “Just What You Need To Know About Variance Swaps”, Sebastien Bossu, Eva Strasser and Regis Guichard 2005.)

Example: variance swaps

Suppose a 1-year variance swap is stuck at 20 with a vega amount of €100,000 (which implies a variance amount of €2,500).

If the index then realises 25% volatility over the next year, the long will receive €562,500 = 2,500 x (25² – 20²).

However if the index only realises 15%, the long will pay €437,500 = 2,500 x (15² – 20²).

So the average exposure for a realised volatility being 5% away from the strike is €500,000 or 5 times the vega amount, as required.
Trading Correlation - Dispersion trades

This section introduces dispersion trades and outlines two types of variance dispersion trade: **vanilla dispersion trades** and **correlation-weighted dispersion trades**. Subsequent sections examine these trades in more detail.

Although correlation is traded most directly through a correlation swap, a correlation exposure will naturally arise when a portfolio of (long/short) options on an index and (short/long) options on constituents of that index is held.

A (long) dispersion trade is a trade which is short index volatility and long volatility on its constituents. Typically such a trade would be done through variance swaps using a **variance dispersion trade**: sell variance on the index, buy variance on its constituents. But it could also be achieved through delta-hedged options, straddles (delta-hedged or not) or any other suitable volatility vehicle. We will concentrate on variance dispersion trades.

A dispersion trade will have exposure to correlation, but also to other factors – for example volatility – depending on the weightings chosen for the constituent legs of the trade. Although a pure exposure to correlation is not achievable through such a static strategy, other exposures can be hedged to some extent by altering the weightings and/or employing a dynamic replication strategy.

The weightings of the single-stock variance swaps are of crucial importance in a dispersion trade. Here we discuss two different weighting schemas: a **vanilla dispersion trade** (the simplest) which has both correlation and outright volatility exposures, and a **correlation-weighted dispersion trade**, whose p/l more closely reflects changes in correlation.

A **vanilla dispersion trade** is one where the variance swaps on the index members are bought in exact proportion to the weights of the members in the index. A long vanilla dispersion trade is short correlation, but it is also long volatility – and the p/l from changes in volatility is generally a more important driver of overall p/l.

A **correlation-weighted dispersion trade** uses implied correlation to weight the single stock variance swaps. Such a dispersion trade will maximise exposure to correlation while ensuring that initial vega exposure is zero. Correlation-weighted dispersion trades have tracked correlation much better than vanilla dispersion trades.

Both types of dispersion trade have historically been profitable. Which type of dispersion trade is preferable depends on desired exposures.

The vanilla dispersion trade has in fact been an efficient way to own volatility, in effect using the alpha available from short correlation to subsidise the cost of being long volatility. On the other hand, the correlation-weighted dispersion trade has profited directly from correlation returns and is less directly affected by moves in volatility.

Below we discuss in detail the drivers of p/l and historical performance of vanilla and correlation-weighted dispersion trades.
Trading correlation - Vanilla dispersion trades

A vanilla dispersion trade is short an index variance swap and long single stock variance swaps, where the vega amount of each single stock swap is proportional to the weight of that stock in the index. The index variance swap vega notional is the sum of single stock vega notional.

A vanilla dispersion trade is short correlation but long volatility. It is long volatility because, with correlation less than one, a given increase in single stock volatilities will lead to a smaller increase in index volatility.

Example: a vanilla dispersion trade is long volatility

We sell €10,000 vega notional of an index variance swap at a strike of 20. We buy a total of €10,000 vega notional of single-stock variance swaps at an average strike of 30. In order to set up a vanilla dispersion trade, the vega notional of each stock will be proportional to its weight in the index.

Using the correlation proxy we estimate correlation at 0.44

\[
\text{Index volatility} = \sqrt{\text{correlation}} \times \text{average single stock volatility}
\]

Suppose that average single-stock volatility increases by 1% whilst correlation stays constant. The correlation proxy tells us the level of index volatility:

\[
\text{Index volatility} = \sqrt{0.44} \times 31% = 20.67%
\]

That is, a 1% increase in single-stock volatilities will lead to only a 0.67% increase in index volatility. Therefore (with correlation less than one) a vanilla dispersion trade (short index volatility, long single-stock volatility) has a positive vega sensitivity. In fact, with correlation constant, the initial vega of the dispersion trade is equal to one minus the square root of the correlation.

This long vega exposure means that the p/l of a vanilla variance dispersion trade has been closely linked with the p/l of a long volatility position (Figure 17). In fact the exposure to correlation has been less important than the exposure to volatility.

Figure 17: The p/l from a vanilla dispersion trade is closely related to long variance p/l

![Figure 17](source)

Figure 18: Vanilla dispersion p/l is positively correlated with long variance p/l

![Figure 18](source)
Figure 18 shows the strong positive correlation between the p/l from a long vanilla dispersion trade and the p/l from a long variance swap on the index. The line of best-fit demonstrates that whilst the variance swap is more leveraged, by a factor of almost 2:1, the dispersion trade p/l has been much more consistently positive – as a result of the alpha earned from being short correlation.

Although the vanilla dispersion trade is technically short correlation, this exposure has been dominated in practice by the long volatility position. In fact there has been a slightly negative correlation between p/l from a long vanilla dispersion trade and moves in correlation (implied minus subsequent realised). This is probably due to the fact that short correlation p/l (implied minus realised) is generally positively correlated with short variance p/l, since similar factors drive both of them. Therefore, the principal exposure of the vanilla dispersion trade – to long volatility – cancels out the simultaneous exposure to short (implied minus realised) correlation.

Despite the fact that a vanilla dispersion trade fails to provide a direct exposure to correlation, it has a number of properties which can make it an attractive asset, especially in comparison to a variance swap.

The advantages of the vanilla dispersion trade in comparison to an outright variance swap are:

1. The short correlation exposure to some extent hedges the long volatility exposure helping to protect against adverse (downwards) moves in volatility and reduce the overall volatility of the trade.
2. The alpha earned from selling correlation (since implied correlation trades at a premium to realised) is used to fund the long volatility position (since volatility also trades at a premium) thus making it a more efficient way of being long volatility.
3. Although the direct correlation exposure is limited, the trade can profit from a decrease in correlation relative to volatility.

At best a vanilla dispersion trade can combine the advantages both from being long volatility and from being short correlation. The short correlation position has historically earned a premium which can offset the cost of entering into a long volatility position. Furthermore the short correlation exposure offers some protection against adverse downward moves in volatility: such decreases in volatility are often accompanied by corresponding reductions in correlation from which the dispersion trade will profit. Although the converse also applies and profits from increases in volatility will be somewhat offset by losses due to the short correlation exposure, the gains from volatility will tend to dominate due to the convexity of the constituent variance swaps.

It should be stressed that whilst a vanilla dispersion trade has historically been an efficient way of owning volatility, this is in large part due to the premium of implied over delivered correlation. If this premium was not present the vanilla dispersion trade would look less attractive. In this scenario such a trade would simply be long volatility with a lesser exposure to short correlation, which would act to reduce both large profits and large losses.

Some market participants view a vanilla dispersion trade as ideal because the long volatility component is seen as a partial hedge against adverse moves in correlation. In fact backtesting shows that the volatility exposure tends to dominate that of correlation, making the position more similar to a subsidised long volatility position.
Trading correlation - Correlation-weighted dispersion trades

It is possible to weight the constituent legs of a variance dispersion trade to make the trade vega-neutral at inception and much more closely related to changes in correlation thereafter. Although the major contributor to the p/l of this trade is the change in correlation, between implied and subsequent realised, over the lifetime of the swap, there are still other significant factors affecting the p/l.

**Vega Exposure:** Ideally, we would like to be able to choose the weights in a variance dispersion trade so that the trade paid out on the basis of the change in correlation whilst remaining independent of volatility. It turns out that this is impossible (at least without a dynamic hedging strategy). We can however choose the weights so that the trade starts off as vega-neutral, although a volatility exposure will develop as correlation subsequently moves. Back-testing shows that the p/l from this trade is closely related to the difference between implied and subsequent realised correlation (Figure 19).

**Weighting:** In order to construct this trade in terms of vega notionals we weight each single-stock variance swap by the weight of the stock in the index multiplied by the implied correlation. We then further multiply by the ratio of the stock’s variance strike to the index variance strike. A variance dispersion trade weighted in this way will be vega-neutral at trade inception and a long position will profit from increases in single stock variances in proportion to the stock’s weight in the index. See discussion below for details. A trade weighted in this way will be referred to as a *correlation-weighted (variance) dispersion trade*.

Note that in practice a simpler weighting scheme for correlation-weighted dispersion trades is often employed. This scheme works by weighting each single-stock variance swap by its index weight multiplied by the square root of the implied correlation. It turns out that this alternative scheme is also vega-neutral at trade inception (Box 5) and results in an almost identical p/l to that described here (see page 25).

**Figure 19:** P/L from a long correlation-weighted dispersion trade is related to correlation p/l

**Figure 20:** … but the relationship between them is non-linear

Source: JPMorgan
Box 2: Dispersion trade weightings

The p/l of a dispersion trade with arbitrary weights \( \alpha_i \) (for the single-stock vega notionals) is given by:

\[
p/l = \frac{K_i^2 - \sigma_i^2}{2K_i} - \sum_i \alpha_i \left( \frac{K_i^2 - \sigma_i^2}{2K_i} \right)
\]

where \( K_i \) and \( \sigma_i \) represent the index and single-stock variance swap strikes; and \( \sigma_I \) and \( \sigma_i \) represent the index and single-stock realised variances.

For a **vanilla dispersion trade** the weights of the vega notionals would be the stock weights: \( \alpha_i = \omega_i \).

For a **correlation-weighted dispersion trade** the weights are given by multiplying the stock weights by the implied correlation and the ratio of stock to index implied volatility: \( \alpha_i = \rho_i \omega_i \frac{K_i}{K} \).

Note that using the alternative weighting \( \alpha_i = \sqrt{\rho_i} \times \omega_i \) achieves a virtually identical effect (see pages 25 and 26 for details).

**Example: a correlation-weighted dispersion trade is initially vega neutral**

Consider our previous example with index volatility at 20%, average single-stock volatility at 30% and correlation at approximately 0.44. We first demonstrate how to set up a correlation-weighted dispersion trade under these circumstances.

For a **vanilla dispersion trade** the weighting of any single-stock variance swap in the trade (in vega-notional terms) would be the weight of the stock in the index. To compute the weighting for the **correlation-weighted dispersion trade** we multiply this stock weight by the correlation (0.44) and then by the ratio of the stock’s volatility to index volatility. Assuming the stock has the average single-stock volatility this will give a weighting factor of 0.44 x (30%/20%) = 0.66 to be multiplied by the stock’s index weight.

Therefore (assuming all stocks have volatilities equal to the average single-stock volatility) we will be short 3 parts index variance and long 2 parts single-stock variance.

Suppose as before that single-stock volatility increases by 1% whilst correlation stays constant. Then according to the previous example, index volatility will increase by 0.66% whilst single-stock volatility will increase by 1%. Since we are short index to single-stock volatility in a 3:2 ratio our net p/l will be zero.

Suppose instead that correlation decreases by 4 correlation points whilst stock volatilities stay constant. Our trade will still be weighted in a 3:2 ratio. Single-stock volatility will remain unchanged, but, using the usual correlation proxy formula, index volatility will increase to 30% multiplied by the square root of 0.40, equal to 19%. Therefore, since we are short index volatility, our trade will make a profit of 1 vega, as a result of the favourable move in correlation.

In the following section we discuss the drivers of p/l for correlation-weighted dispersion trades.
Drivers of p/l for a correlation-weighted dispersion trade

It turns out that if you are long a correlation-weighted dispersion trade you will want the following to occur:

1. Correlation to decrease.
2. Average single-stock volatility to increase - if correlation has decreased. (If correlation has increased you would want average single-stock volatility to decrease in order to minimise your losses.)
3. Dispersion of stock volatilities within the index to increase.

In the following paragraphs we explain why the p/l is driven by these factors and how their importance varies under differing volatility conditions. Box 3 and Box 4 below summarise the maths.

1. Correlation is the major driver for correlation-weighted dispersion trade p/l

The major driver for p/l of a correlation-weighted variance dispersion trade is correlation (Figure 21). That is, the returns from a long correlation-weighted dispersion trade are closely related to the difference between the implied correlation at trade inception and the subsequent realised correlation (Figure 19). Note that a long correlation-weighted dispersion trade is short correlation: if you have sold index variance against single-stock variance you want correlation to decrease.

Terminology: In the following discussion we use the term correlation p/l to refer to the difference between implied correlation and subsequent realised correlation over the appropriate term. Whilst this is really a hypothetical p/l, as it is not possible to trade this directly, it represents the difference between the correlation priced in by the market and that actually delivered. In this sense it is the target p/l of a correlation-weighted dispersion trade (although the latter is expressed in vegas rather than correlation points).

The correlation between correlation-weighted dispersion trade returns and long volatility returns is almost zero: indeed slightly negative (Figure 22). That is, unlike the vanilla dispersion trade, the correlation-weighted dispersion trade is strongly exposed to changes in correlation but roughly neutral to changes in volatility.

Figure 21: There is a strong relationship between p/l and correlation p/l...

Figure 22: …but no clear relationship between p/l and volatility p/l

Source: JPMorgan
Source: JPMorgan
Although there is a strong link between p/l from a correlation-weighted dispersion trade and correlation p/l, the relationship is non linear (Figure 21). This is due to the interplay between the correlation p/l and scaling factor described in the following section.

2. The level of exposure to correlation is scaled according to prevailing volatility conditions

In a long correlation-weighted dispersion trade, the level of exposure to the hypothetical correlation p/l will be affected by the realised average single-stock volatility and the index implied volatility. To compute an estimate of the p/l (at least of that part of the p/l driven by changes in correlation) we multiply the correlation p/l by a scaling factor equal to the average realised single-stock volatility squared divided by twice the index variance-swap strike. From a p/l viewpoint the index variance-swap strike is a constant, so the scaling factor has the effect of making the trade long volatility when correlation decreases and short volatility when correlation increases. Put another way: at higher volatilities this scaling factor causes the p/l resulting from both increases and decreases in correlation to be magnified.

This effect is demonstrated by plotting the returns from the correlation-weighted dispersion trade against the average realised volatility of the stocks in the index (Figure 23). As average volatility increases, the spread of returns from the correlation-weighted dispersion trades widens. Due to the alpha earned from short correlation, no correlation-weighted dispersion trade made a (significant) outright loss unless average single-stock realised volatility was over 40%.

Figure 23: p/l from a correlation-weighted dispersion trade is magnified at higher volatilities

![Figure 23](image)

Source: JPMorgan

Figure 24: p/l is relatively well approximated by multiplying the scaling factor and the correlation p/l

![Figure 24](image)

Source: JPMorgan

The scaling factor by which the change in correlation is multiplied is such that correlation returns will be magnified most in times of high realised volatility, particularly when implied volatility underestimates the future realised volatility. Conversely correlation returns will be attenuated when realised volatility is low, especially when implied volatility has overestimated that subsequent realised. This explains why, in the current environment with low and generally declining volatility, consistently overestimated by implied levels, we have had very modest returns from long correlation-weighted dispersion trades, but high hypothetical correlation p/l (see page 28 for a discussion of returns).

Figure 24 demonstrates that multiplying the hypothetical correlation p/l by the scaling factor closely approximates the p/l of a correlation-dispersion trade.

Due to the correlation between correlation and volatility, the scaling factor described above turns out to be negatively correlated with correlation p/l. The same factors which cause implied correlation to be large with respect to realised correlation (leading to positive correlation p/l) also tend to cause implied volatility to be large with respect to realised volatility, leading to a small scaling factor and hence smaller than expected p/l from the correlation dispersion trade. Conversely if correlation p/l is negative, due to larger realised correlation than
implied, this would normally be accompanied by realised volatility being greater than that implied and hence a large scaling factor, magnifying the losses from the short correlation position. This accounts for the negative convexity observed in Figure 20: as correlation p/l increases, the scaling factor will tend to decrease, reducing the correlation gains.

**Example A: High scaling factor**

On July 1st 2001, implied index volatility was 25% and implied correlation 0.59. Over the following six months the index realised 31% volatility, with average realised single-stock volatility at 43.5% and a realised correlation of 0.49. Thus correlation p/l was 0.59-0.49=0.10, and the scaling factor was (43.5%^2)/(2 x 25%) = 0.38, giving an estimated 6-month correlation-weighted dispersion trade p/l of 3.8 vegas (the true p/l was 4.1). Note that in this case everything moved in our favour: correlation went down to give us a positive p/l and volatility went up to give a relatively large scaling factor.

**Example B: Low scaling factor**

On October 20th 2003, implied index volatility was 25.6% with implied correlation at 0.73. Six months later, realised index volatility was 14.7% with average realised single-stock volatility at 22.5% and realised correlation of 0.41. In this case correlation p/l was 32 correlation points, but the scaling factor was only 0.1 giving an estimated 6-month correlation-weighted dispersion trade p/l of 3.2 vegas (the true p/l was 3.3). In this case the correlation p/l was much higher than in our first example, but volatility moved against us to produce a very small scaling factor which heavily reduced our p/l.

This is quite characteristic of the recent performance of the correlation-weighted dispersion trade: whilst correlation p/l has been high, volatility has been low, with realised volatility less than that implied, leading to a small scaling factor and reduced p/l (see Figure 19).

The above examples and Figure 24 show that whilst correlation-weighted dispersion p/l is well approximated by multiplying the hypothetical correlation p/l by the scaling factor, there are clearly other drivers influencing the p/l. Box 3 and

Box 4 give the full picture: correlation-weighted dispersion p/l depends on both hypothetical correlation p/l and *volatility dispersion* p/l with associated scaling factors for each. The following section discusses this p/l driver.
Box 3: The p/l components of correlation-weighted dispersion

The p/l for a correlation-weighted dispersion trade can be written as follows:

\[ p/l = \text{factor1} \times \text{correlation p/l} + \text{factor2} \times \text{volatility dispersion p/l} \]

where

- \( \text{correlation p/l} = \text{implied correlation} - \text{realised correlation} \)
- \( \text{volatility dispersion p/l} = \text{variance(realised stock volatilities)} - \text{variance(implied stock volatilities)} \)

and

\[ \text{factor1} = \frac{\left(\text{average stock volatility}\right)^2}{2 \times \text{index strike}} \]
\[ \text{factor2} = \frac{\text{implied correlation}}{2 \times \text{index strike}} \]

or more formally:

\[
p/l = \frac{\left(\sum \omega_i \sigma_i\right)^2}{2K_i} \left(\rho_i - \rho_H\right) + \frac{\rho_i}{2K_i} \left[\sum \omega_i \left(\sigma_i - \sum \omega_j \sigma_j\right)^2 - \left(K_i - \sum \omega_j k_j\right)^2\right]
\]

The only approximation in the above equation is due the error introduced by using the proxies to calculate implied and realised correlations. In practice for any reasonable index the p/l as expressed above will be virtually identical to the true p/l of a correlation-weighted dispersion trade.

3. Volatility Dispersion: an additional driver of correlation-weighted dispersion trade p/l

As indicated above, the estimate for the correlation-weighted dispersion trade p/l given by multiplying the correlation p/l by the scaling factor is not entirely accurate. The error is principally accounted for by a second component affecting the trade’s p/l known as the volatility dispersion. This is a measure of the change in dispersion of the single-stock volatilities within the index. A long dispersion trade will profit if the single-stock volatilities become more spread out (dispersed) with a mixture of very high and very low volatility stocks, and will lose if the volatilities become more similar.

Volatility dispersion is usually a small component of the correlation-weighted dispersion trade p/l but can become important in periods of high and fluctuating volatility (Figure 25). This is especially true when a particular stock undergoes a very large change (typically an increase) in volatility, such as happened to Ahold in February 2003. For 3-month correlation-weighted dispersion trades begun in January 2003, the average p/l was 12%. Volatility dispersion was responsible for about 4.5% (over one-third) of this figure.

As is the case with volatility and correlation, it is possible to measure both implied and realised volatility dispersion. The implied volatility dispersion computes the weighted variance of the stock implied volatilities across the index whereas the realised volatility dispersion does the same thing for realised volatilities.

Volatility dispersion p/l is then defined as the difference between the implied and subsequent realised volatility dispersion. It will be positive when the spread of volatilities across the index increases during the course of the trade and negative when it decreases. Since it is measured as a variance of stock volatilities, it increases most dramatically when one, or a few, stocks undergo very large changes in volatility compared to when the volatilities of the component stocks change more uniformly across the index. The volatility dispersion is multiplied by a scaling factor, of half the ratio of implied correlation to index variance strike, when computing its contribution to the p/l of a correlation-weighted dispersion trade (see Box 3 and Box 4 for details).
Although there is no actual market for implied volatility dispersion, it is interesting to note that it has consistently traded above that delivered, rarely slipping below (Figure 26). This can be interpreted as the market predicting that stock volatilities will move relative to each other more than they actually do.

In the following section we investigate volatility dispersion further. We first illustrate that without the volatility dispersion term the product of the scaling factor and the correlation p/l provides a reasonable estimate of correlation-weighted dispersion trade p/l (Figure 27). Most of the error in this approximation is due to the failure to take the volatility dispersion component into account. If we estimate the correlation-weighted dispersion trade p/l more accurately, by also considering the volatility dispersion p/l and its associated scaling factor, then the resulting estimate of p/l is very close to the true value (Figure 28). The only error in this final approximation should be due to the approximation introduced by the using the proxy to estimate correlation.
The p/l of a (long) correlation-weighted variance dispersion trade is given by:

\[
\frac{1}{2K_i} \left[ \left(K_i^2 - \sigma_i^2 \right) - \rho_i \sum \omega_i \left(K_i^2 - \sigma_i^2 \right) \right]
\]

where \( \rho_i \) is the implied correlation at trade inception.

Using the standard approximations \( \rho_{ii} \approx \frac{\sigma_i^2}{\left( \sum \omega_i \sigma_i \right)^2} \) and \( \rho_i \approx \frac{K_i^2}{\left( \sum \omega_i K_i \right)^2} \) we can express the p/l as:

\[
p/l = \frac{1}{2K_i} \left[ \left( \rho_i - \rho_{ii} \right) \left( \sum \omega_i \sigma_i \right)^2 + \rho_i \sum \omega_i \left( \sigma_i - \sum_j \omega_j \sigma_j \right)^2 \left( K_i - \sum_j \omega_j k_j \right)^2 \right]
\]

This p/l consists of two parts. The first part: \( \left( \rho_i - \rho_{ii} \right) \frac{\left( \sum \omega_i \sigma_i \right)^2}{2K_i} \) represents the change in correlation scaled by the ratio of average single-stock realised volatility squared to twice the index variance swap strike.

The second part: \( \frac{\rho_i}{2K_i} \sum \omega_i \left( \sigma_i - \sum_j \omega_j \sigma_j \right)^2 \left( K_i - \sum_j \omega_j k_j \right)^2 \) represents the change in the dispersion of the index volatilities. We refer to the value \( \sum \omega_i \left( K_i - \sum_j \omega_j K_j \right)^2 \) as the implied volatility dispersion and the value \( \sum \omega_i \left( \sigma_i - \sum_j \omega_j \sigma_j \right)^2 \) as the realised volatility dispersion. This second value is the weighted standard deviation of the single stock volatilities within the index. Its minimum is zero and will be attained when all volatilities are identical. Note that a long correlation weighted dispersion trade will be long volatility dispersion.
Volatility dispersion and stock-specific events

A (long) correlation-weighted dispersion trade will make money when the p/l from the portfolio of single-stock variance swaps exceeds the p/l from the index variance swap. This can happen in two ways:

1. Single stock volatility increases with respect to index volatility (i.e. correlation decreases)

2. One (or a few) of the single-stock volatilities increase by large amounts. The convexity of the variance swap payoff means that the large increase(s) in single-stock variance swap p/l more than offsets the extra p/l in the index variance swap, where the volatility gains are averaged across the index (positive volatility dispersion).

Usually the first of these factors dominates, but in some circumstances the second factor – volatility dispersion – can become very important. This is a result of the variance swap convexity. For small changes in volatility the variance swap p/l is very close to the difference between the implied and realised volatility, but if volatility increases dramatically the long variance swap position will profit in excess of what would be predicted by a linear payoff.

Example: effect of convexity on dispersion p/l

Consider a long variance swap position struck at 20. If subsequent realised volatility is 21, the variance swap p/l will be 1.03 vegas – almost what would be expected from a linear payoff. If instead the realised volatility is 40 the p/l will be 30 vegas (in contrast to the 20 expected from a linear payoff), and if realised volatility is 100 the p/l will be 240 vegas: three times that expected from a linear payoff.

Thus if a single stock in the index undergoes a huge increase in volatility, the extra p/l generated by the convexity of a long variance swap position can have a significant effect on the whole trade.

Suppose we have an index consisting of 50 equally weighted stocks, each with implied volatility of 20%. The index has implied volatility of 14% so correlation is 0.5. Suppose that we set up a correlation-weighted dispersion trade, and at the end of the trade, correlation has remained constant, one stock has realised 100% volatility and all others have realised only 20%.

The resulting increase in single stock volatility with correlation constant will lead to a small increase in index volatility and a corresponding small loss on the index leg of the trade of about 1.2 vegas. For 49 of the fifty single-stock variance swaps the p/l is zero. For the remaining single-stock variance swap, due to the convexity effect, the p/l will be 240 vegas. The weighting scheme determines that the notional of this variance swap will be 1.4% of the index variance swap notional. Therefore the single-stock variance swap leg will make a profit of 3.4 vegas compared to the 1.2 vegas loss on the index leg, leading to an overall profit of 2.2 vegas.

Consider what happened in the above example: overall correlation did not change; the correlation p/l was zero. But, the correlation-weighted dispersion trade still made a profit of 2.2 vegas. This is due entirely to convexity of the variance swap payoff for the stock experiencing the large increase in volatility. Without this convexity, the profit generated by the single stock leg would have been completely balanced by a corresponding rise in index volatility and a loss on the index leg. The extra p/l generated by the convexity of the variance swap payoffs is exactly what we described above as the volatility dispersion p/l.
This is almost precisely what happened when Ahold sold off sharply in February 2003. The effect is can be most easily seen on a 3-month time horizon. On February 24th the stock price plummeted from €8.20 to €3.04, a price change of about 63% (Figure 29). For the three months prior to this date, the 3-month implied volatility had been trading between about 50 and 60%.

For the three months following this event Ahold’s 3-month realised volatility was between 200 and 220% – mostly due to this one huge price change. Therefore, assuming an implied volatility of 50% and a realised volatility of 200%, a long variance swap would have made 375 vegas, in contrast to the linear predicted vega p/l of 150.

Even though Ahold had a weight of only about 0.9% in the index before this event, the extra p/l generated from being long the variance swap contributed about 3 vegas to the correlation-weighted dispersion trade p/l, in the form of volatility dispersion p/l (Figure 30).

Note that in theory an extreme move in one stock could actually reduce correlation if it doesn’t drag other stocks with it. So a dispersion trade is what the name suggests: it profits directly from the returns and volatilities of the stocks in the index becoming more dispersed over time.
Alternative weighting schema and dynamic replication

The condition for a dispersion trade to be initially vega neutral is that the sum of the vega-notionals of the single stocks is equal to the square root of the index implied correlation (see Box 5). For a correlation-weighted dispersion trade, this is achieved by multiplying the stock weight by the implied correlation and then by the ratio of the stock implied volatility to index implied volatility. A more straight-forward means of achieving this is simply to multiply each stock weight by the square root of the implied correlation. As the stock weights sum to one, our condition is automatically fulfilled. Moreover, the single-stock vega amounts are held in relation to each other according only to their weight in the index.

This weighting scheme does not have such easily analysed properties as the previously described correlation-weighted dispersion trade, but in some senses it is simpler to implement – we only have one weighting parameter to control. Backtesting shows that the p/l from this weighting scheme is virtually identical to that of the correlation-weighted dispersion trade, and this simpler scheme is generally the one used in practice.

Figure 31: p/l from the new weighting scheme tracks correlation-weighted dispersion p/l almost perfectly (6m maturity)

Figure 32: The same at 1-year maturity

In view of such close correlation between the two weighting schemes, we conclude that it doesn’t matter which is used: the p/l will be extremely similar and the drivers can be considered the same. Therefore the preceding analysis of correlation-weighted dispersion trades can be taken to apply to both weighing schemas.

Dynamic replication strategies

All of the above discussion refers to static strategies. By weighting the single-stock variance swaps by the correlation we construct an instantaneous hedge to make the trade insensitive to changes in volatility. However, as correlation increases or decreases this hedge breaks down and the trade takes on a vega-exposure.

As correlation increases we must buy vega to hedge the developing short vega exposure. We can do this by increasing the weighting of the single-stock legs, or by buying back some of the short index variance position. Similarly as correlation decreases we become long vega, and must sell it, either by reducing our single-stock variance holdings or increasing our short index variance exposure. In this way the trade can remain vega neutral throughout and the p/l will be equal to the correlation p/l (implied correlation minus subsequent realised correlation) and second order terms – such as the volatility dispersion p/l.

Note that this hedging strategy works by effectively eliminating any vega exposure developing as a result of the scaling factor by which the correlation p/l is multiplied. However the exposure to volatility dispersion still remains, so even with this strategy there will not necessarily be a direct linear relation between the resulting p/l and the difference between implied and subsequent realised correlation.
Box 5: Vega sensitivity and weightings of dispersion trades

Suppose we have a long position in a variance dispersion trade, with the constituents parts of the trade expressed in vega-notional terms and with each single-stock variance swap weighted by some value $\alpha_i$. The payoff of this trade can then be expressed as

$$\begin{align*}
p/l &= \frac{K_i^2 - \sigma_i^2}{2K_i} - \sum_i \alpha_i \left( \frac{K_i^2 - \sigma_i^2}{2K_i} \right).
\end{align*}$$

Suppose that over the lifetime of the trade, the correlation $\rho$ is equal to the implied correlation at trade inception, and suppose also that volatility increases uniformly across the index in that the realised volatility $\sigma_i$ of each constituent is equal to its implied volatility $K_i$ plus a small constant $\varepsilon$. Then:

$$\begin{align*}
\sigma_i^2 &\approx \rho \left( \sum \omega_i \sigma_i \right)^2 = \rho \left( \sum \omega_i (K_i + \varepsilon) \right)^2 = \rho \left( \varepsilon + \sum \omega_i K_i \right)^2 \approx \rho \varepsilon^2 + 2 \rho \varepsilon \sum \omega_i K_i + K_i^2, \quad \text{and} \\
p/l &= \frac{K_i^2 - \sigma_i^2}{2K_i} - \sum_i \alpha_i \left( \frac{K_i^2 - \sigma_i^2}{2K_i} \right) = -\frac{\rho \varepsilon^2 - 2 \rho \varepsilon \sum \omega_i K_i}{2K_i} + \sum_i \alpha_i \left( \frac{2K_i \varepsilon + \varepsilon^2}{2K_i} \right) \\
&\approx -\frac{\rho \varepsilon \sum \omega_i K_i}{K_i} + \sum_i \alpha_i \varepsilon = \varepsilon \left[ \sum \alpha_i - \frac{K_i}{\sum \omega_i K_i} \right] = \varepsilon \left[ \sum \alpha_i - \sqrt{\rho} \right].
\end{align*}$$

Therefore the trade will be vega-neutral exactly when $\sqrt{\rho}$ is equal to $\sum \alpha_i$. Thus for a vanilla dispersion trade with $\alpha_i = \omega_i$ we will be long vega since $\sum \alpha_i = \sum \omega_i = 1$ and correlation will be less than one.

If we set $\alpha_i = \rho \omega_i \frac{K_i}{K_i}$ then $\sum \alpha_i = \rho \frac{\sum \omega_i K_i}{K_i} = \rho \frac{K_i}{\sqrt{\rho}} = \sqrt{\rho}$ and the trade is vega-neutral.

Furthermore the p/l from the trade weighted in this way is

$$\begin{align*}
p/l &= \frac{K_i^2 - \sigma_i^2}{2K_i} - \sum_i \alpha_i \left( \frac{K_i^2 - \sigma_i^2}{2K_i} \right) = \frac{K_i^2 - \sigma_i^2}{2K_i} - \rho \sum \omega_i K_i \left( \frac{K_i^2 - \sigma_i^2}{2K_i} \right) \\
&= \frac{1}{2K_i} \left[ (K_i^2 - \sigma_i^2) - \rho \sum \omega_i (K_i^2 - \sigma_i^2) \right].
\end{align*}$$

That is a long dispersion position profits from increases in variances in single stocks in proportion to their weight in the index.

Alternatively we can choose weights of $\alpha_i = \omega_i \sqrt{\rho}$ for the single stock vega-notionals. Since this preserves the relationship $\sum \alpha_i = \sqrt{\rho}$ the trade is initially vega neutral. Backtesting shows that the p/l from a dispersion trade using this weighting is virtually identical to the p/l achieving using the more complex weighting: $\alpha_i = \rho \omega_i \frac{K_i}{K_i}$. 


Risks and returns of trading correlation

Returns from short correlation trades have been almost consistently positive over the last five years, be they vanilla dispersion or correlation-weighted dispersion trades (Figure 33). For example, selling 6-month correlation via correlation-weighted dispersion has generated an average annual return of 2.9 vegas, assuming a 3 vega bid-offer spread (Table 2). These results reflect the overall premium of implied correlation over realised correlation, which is what dispersion trades seek to capture. Any losses have tended to occur only in times of high market volatility, although even then short correlation has tended to outperform outright short volatility positions.

Table 2: Correlation returns: annualised returns and risks (in vegas) from dispersion trades

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<td><strong>Long vanilla dispersion</strong></td>
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<tr>
<td>Return ( vegas, annualised)</td>
<td>8.6%</td>
<td>4.8%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Volatility of return (annualised)</td>
<td>4.5%</td>
<td>2.0%</td>
<td>1.3%</td>
</tr>
<tr>
<td>Risk-Return</td>
<td>1.9</td>
<td>2.4</td>
<td>2.4</td>
</tr>
<tr>
<td><strong>Long correlation-weighted dispersion</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return ( vegas, annualised)</td>
<td>8.8%</td>
<td>2.9%</td>
<td>1.4%</td>
</tr>
<tr>
<td>Volatility of return (annualised)</td>
<td>3.4%</td>
<td>1.3%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Risk-Return</td>
<td>2.6</td>
<td>2.2</td>
<td>2.6</td>
</tr>
<tr>
<td><strong>Short index variance (for comparison)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return ( vegas, annualised)</td>
<td>3.8%</td>
<td>-0.1%</td>
<td>-2.2%</td>
</tr>
<tr>
<td>Volatility of return (annualised)</td>
<td>9.9%</td>
<td>5.4%</td>
<td>2.7%</td>
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<tr>
<td>Risk-Return</td>
<td>0.4</td>
<td>0.0</td>
<td>-0.8</td>
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</table>

Source: JPMorgan. Results assume 3% (3 vega) bid-offer on dispersion trades and 1.5% bid-offer on variance swaps. Data August 2000 to date.

Risk-return figures for dispersion trades have been good, with annualised risk-return ratios around 2.0 – 2.5 and stable across maturities (Table 2). This corresponds closely to correlation risk-returns – defined as the risk-return on the difference between implied correlation and that subsequently realised. Note that risk-return ratios for vanilla dispersion trades are similar to correlation-weighted dispersion trades (with similarly modest returns since mid 2003) but with vanilla dispersion the underlying exposure is more to volatility than it is to correlation.

Figure 33: p/l from correlation-weighted dispersion trades at various maturities

Figure 34: Implied minus subsequent realised correlation, various maturities

Annualised returns are highest for the short-dated trades, but the risks here are greater too. The p/l generated by a 3-month trade over 3 months is similar to that generated by a 12-month trade over a year, so rolling four 3-month trades over a year has been much more profitable than simply holding a single 1-year trade. However, the returns from the 3-month trades are much more volatile, and when risk-return ratios considered, these two trends largely cancel each other out. The risk-returns for these trades are similar across the maturities.
Over the last two years, implied correlation has continued to trade substantially above the level of that being delivered (Figure 34). However this spread has not been well represented by the modest returns seen from correlation-weighted dispersion trades over the same period (Figure 33). This is due to the declining volatility environment.

Recall that the principal driver of p/l for a correlation-weighted dispersion trade is the difference between implied and subsequent delivered correlation multiplied by a scaling factor. This scaling factor will be small when realised volatility is low and especially when it is also below that implied. The scaling factor explains why in the current low volatility environment, returns from a long correlation-weighted dispersion trade have been very modest when compared to the large premium of implied correlation over that subsequently realised.

Table 3 summarises the performance of the two types of dispersion trade since January 2003, with short variance performance included for comparison. To reflect the increasing liquidity of dispersion trades we have assumed a bid-offer spread of 2.5% compared to the 3% spread assumed for the entire period since August 2000 in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>3m</th>
<th>6m</th>
<th>1y</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Long vanilla dispersion</strong></td>
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<tr>
<td>Return (vegas, annualised)</td>
<td>2.5%</td>
<td>0.0%</td>
<td>-1.1%</td>
</tr>
<tr>
<td>Volatility of return (annualised)</td>
<td>3.3%</td>
<td>1.1%</td>
<td>0.3%</td>
</tr>
<tr>
<td>Risk-Return</td>
<td>0.0</td>
<td>0</td>
<td>-3.5</td>
</tr>
<tr>
<td><strong>Long correlation-weighted dispersion</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return (vegas, annualised)</td>
<td>6.2%</td>
<td>1.9%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Volatility of return (annualised)</td>
<td>3.1%</td>
<td>0.9%</td>
<td>0.2%</td>
</tr>
<tr>
<td>Risk-Return</td>
<td>2.0</td>
<td>2.2</td>
<td>2.1</td>
</tr>
<tr>
<td><strong>Short index variance (for comparison)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Return (vegas, annualised)</td>
<td>19.9%</td>
<td>13.1%</td>
<td>7.0%</td>
</tr>
<tr>
<td>Volatility of return (annualised)</td>
<td>2.9%</td>
<td>1.4%</td>
<td>0.6%</td>
</tr>
<tr>
<td>Risk-Return</td>
<td>6.8</td>
<td>9.7</td>
<td>12.0</td>
</tr>
</tbody>
</table>

Source: JPMorgan. Results assume 2.5% (2.5 vega) bid-offer on dispersion trades and 1.5% bid-offer on variance swaps. Data January 2003 to date.

Since early 2003, absolute levels of returns from dispersion trades (both vanilla and correlation-weighted) have been markedly reduced. Both of these types of trade have suffered from the reduction in levels of volatility: vanilla trades through their outright long volatility exposure and correlation-weighted trades through the resulting scaling down of correlation premium.

The vanilla dispersion trade has suffered the most: assuming prevailing bid/offers of around 2.5%, returns have been just positive at 3-month maturities, zero at 6-months and negative at 1 year; overall risk-return ratios are much lower than corresponding figures for the entire period since 2000.

**Correlation-weighted dispersion trades have fared better:** returns have remained positive at all maturities and with the corresponding reduction in the volatility of the strategy, they have even managed to preserve the risk-return ratios of around 2.

**Looking forward …**

Going forward, **correlation-weighted dispersion trades** may continue to be profitable, particularly if realised correlation falls towards levels observed ten years ago in the previous low-volatility regime. Alternatively, **long vanilla dispersion trades** may provide an efficient way to own volatility. Finally, the increasing liquidity available in the correlation swap market, coupled with generally lower levels of implied correlation compared to dispersion trades, may make **correlation swaps** the most attractive vehicle for owning correlation.
What are the risks of trading correlation?

Correlation moves with volatility: Factors which cause volatility to increase tend also to lead to an increase in correlation. So whilst a correlation swap or correlation-weighted dispersion trade (at least initially) is technically vega-neutral, if volatility were to increase, it is likely that correlation would also increase.

In this case, a long (un-hedged) correlation-weighted dispersion trade, which is short correlation, would become short vega also and the losses would be magnified. In the same way a decrease in correlation would most likely be accompanied by a decrease in volatility which would act to limit the potential gains.

This accounts for the negative convexity of the graph of correlation-weighted dispersion p/l against correlation p/l as observed in Figure 20 and perhaps indicates that a correlation swap strike should even trade higher than index implied correlation.

Change of volatility environment: Of course it is possible for correlation, especially realised correlation, to increase without volatility increasing. Over the long term this seems to be exactly what has happened – correlation is currently significantly higher than it was ten years ago even though volatility is at similar levels. This observation is backed up by the fact that correlation between indices has also been increasing.

If the volatility environment were to change so that stock movements are driven more by top-down macro factors as opposed to stock or sector-specific factors such as M&A activity, then correlation could increase even without an accompanying increase in volatility. This suggests that a long-term view of correlation could be quite different from a long-term view of volatility, whereas short term views on correlation and volatility are likely to be more similar.

Mark-to-market risk: If a correlation trade is closed out before expiry, or if forward correlation is traded, there is then an exposure to future implied levels of correlation as well as, or instead of, realised correlation. In early stages of a correlation trade the exposure to prevailing levels of implied correlation will be significant, but as expiry approaches the mark-to-market p/l will increasingly reflect the realised correlation to date.

Reconstitution risk: When a dispersion trade is set up the volatilities of the current index constituents are traded against the volatility of the index. Over time it is possible that the index members (and/or their weightings in the index) will change.

Index members may be demoted, may be bought out by other companies, or may move their listing elsewhere (as in the case of Royal Dutch/Shell). In this situation the basket of single-stock variance swaps held may cease to be representative of the index and, without re-weighting, the hedge against volatility will partially break down. Note that this risk will increase with the maturity of the trade.
Correlation across sectors and indices

Correlation across indices

The realised correlation of an index can be thought of as a measure of diversification within that index. This diversification could be across different stocks in a sector, different sectors, or even different countries. Whilst there is no a priori reason why a larger index should have lower correlation, the fact that this does happen in practice should not be surprising. The larger indices such as the STOXX 600, S&P500 are much more diversified compared to indices such as the DAX, CAC and AEX which contain between only 25 and 30 members. However, there are exceptions to this rule. Two of these are indicated below:

1. The Dow Jones Global Titans index. This has only 50 members but very low correlation: the index is spread across a number of different economies (US, UK, Europe, Japan) so a lower correlation would be expected.

2. The Nasdaq index. This has 100 members but historically has had a high correlation. This is due to the index’s concentration in the technology sector, meaning that the index has less sector diversification than other indices of comparable size.

Figure 35: Over the past year correlation has come down uniformly across a range of indices

Figure 36: There is a negative correlation between (log) index size and 6-month realised correlation

Inter-Sector and Intra-Sector Correlation

Just as we can model volatility by breaking it down into market volatility, sector-specific volatility and stock-specific volatility, we can break down the correlation of a diversified index into correlation between sectors (inter-sector correlation) and correlation of stocks within a sector (intra-sector correlation).

Inter-sector correlation is computed by considering an index to be made up of its constituent sector indices and calculating the correlation of the returns of these sectors. Intra-sector correlation is simply the average of the realised correlations of stocks within the sub-indices.

If the correlations of all the sub-indices are identical, then it will be the case that overall index correlation is a product of inter-sector correlation and intra-sector correlation. In practice, index correlation is somewhat greater than the product of intra- and inter-sector correlation, reflecting differing correlations within the sectors (Figure 37).
Of the 10 major sectors of the STOXX 600, Energy has by far the largest correlation averaging around 70% over the past five years. This might be expected since energy companies are generally highly correlated with the oil price and hence with each other. The technology and telecoms sectors have also been consistently highly correlated compared to other sectors, especially in the two years prior to mid 2002 when the TMT bubble burst. The least correlated sectors have tended to be industrials, non-cyclical goods and more recently healthcare, where stocks prices have been driven more by idiosyncratic risk.

If index correlation is low this means that most volatility is generated bottom-up from single-stock volatility. In such an environment there is scope for stock-picking strategies to perform well. In a high-correlation environment stock picking gains will be more elusive (and investors may do just as well by index-tracking). If inter-sector correlation is low, this means that there is scope to outperform (or underperform) through sector-allocation.

However if inter-sector correlation is high but intra-sector correlation low, opportunities are more likely to arise from differentiating the stocks within each sector. The above discussion suggests that in the STOXX600, the most promising sectors for stock-picking are non-cyclicals, industrials and healthcare, whilst the sector offering the least opportunity is Energy, followed by Technology, Telecoms and Utilities.

For indices such as the Global Titans (and to a lesser extent the Euro Stoxx 50) which are split across a number of countries it would also be possible to break index correlation down into inter-country and intra-country correlation.

**Correlation between indices**

One of the characteristics of the current low volatility environment is that whilst average single-stock volatilities are low, and index volatility is also relatively low, stock correlation is quite high. Typically correlation has tended to rise and fall with levels of volatility, but recently although correlation has fallen, it is still high relative to the low level of volatility.

There are two differing explanations for this. The first is that correlation is in some sense currently lagging behind volatility, and due to fall further, taking index volatility lower with it. The second explanation is that levels of correlation are inherently greater now than they were ten years ago. Possible reasons for this include increasing globalisation, effects of European Union, greater transparency and faster dissemination of information from electronic trading.
In an attempt to quantify the effect of increasing globalisation, we looked at the correlations between a number of major worldwide indices. We considered the correlation between the FTSE and the S&P since 1980 as well as Euro Stoxx 50 – FTSE correlation since 1990. In order to produce a single series summarising global inter-index correlation, we calculated the average pairwise correlation between a basket of ten major indices (Figure 39). The indices included in this basket were Euro Stoxx 50, FTSE, CAC, IBEX, AEX, SMI S&P500, Nasdaq, Nikkei and Hang Seng.

From the results it is clear that these correlations have tended to increase over time. Indeed, looked at over a 25-year perspective, the current levels of correlation, although lower than a few years ago appear to be very high when compared with their history. Note the large spike in correlation resulting from the crash of October 1987.

**Figure 39: Average 1-year pairwise correlation between a basket of major global indices**

**Figure 40: 1-year pairwise correlation between Euro Stoxx 50 and FTSE 100**

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